South Sudan

Secondary Mathematics 1

Secondary Mathematics has been written and developed by Ministry of General Education and Instruction, Government of South Sudan in conjunction with Subjects experts. This course book provides a fun and practical approach to the subject of mathematics, and at the same time imparting life long skills to the pupils.

The book comprehensively covers the Secondary 1 syllabus as developed by **Ministry of General Education and Instruction.**

Each year comprises of a Student's Book and Teacher's Guide.

The Teacher's Guides provide:

- Full coverage of the national syllabus.
- A strong grounding in the basics of mathematics.
- Clear presentation and explanation of learning points.
- A wide variety of practice exercises, often showing how mathematics can be applied to real-life situations.
- It provides opportunities for collaboration through group work activities.
- Stimulating illustrations.

All the courses in this primary series were developed by the Ministry of General Education and Instruction, Republic of South Sudan. The books have been designed to meet the primary school syllabus, and at the same time equiping the pupils with skills to fit in the modern day global society.

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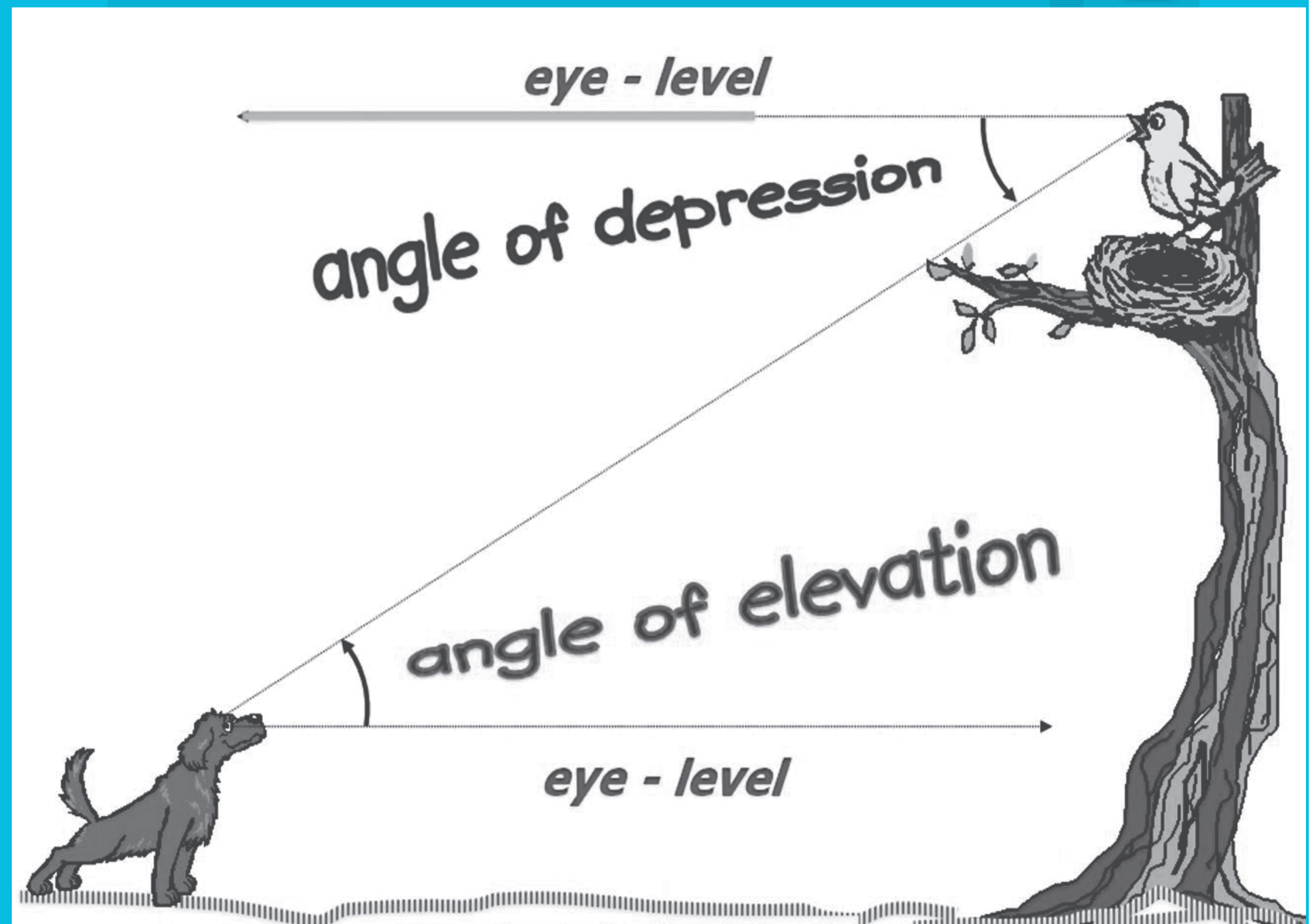
Secondary Mathematics 1 Teacher's Guide

South Sudan

Secondary Mathematics 1
Teacher's Guide



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South Sudan

SECONDARY
1

Mathematics

Teacher's Guide 1

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FOREWORD

I am delighted to present to you this Teacher's Guide, which is developed by the Ministry of General Education and Instruction based on the new South Sudan National Curriculum. The National Curriculum is a learner-centered curriculum that aims to meet the needs and aspirations of the new nation. In particular, it aims to develop (a) Good citizens; (b) successful lifelong learners; (c) creative, active and productive individuals; and (d) Environmentally responsible members of our society. This textbook, like many others, has been designed to contribute to achievement of these noble aims. It has been revised thoroughly by our Subject Panels, is deemed to be fit for the purpose and has been recommended to me for approval. Therefore, I hereby grant my approval. This Teacher's Guide shall be used to facilitate learning for learners in all schools of the Republic of South Sudan, except international schools, with effect from 4th February, 2019.

I am deeply grateful to the staff of the Ministry of General Education and Instruction, especially Mr Michael Lopuke Lotyam Longolio, the Undersecretary of the Ministry, the staff of the Curriculum Development Centre, under the supervision of Mr Omot Okony Olok, the Director General for Quality Assurance and Standards, the Subject Panelists, the Curriculum Foundation (UK), under the able leadership of Dr Brian Male, for providing professional guidance throughout the process of the development of National Curriculum, school textbooks and Teachers' Guides for the Republic of South Sudan since 2013. I wish to thank UNICEF South Sudan for managing the project funded by the Global Partnership in Education so well and funding the development of the National Curriculum, the new textbooks and Teachers' Guides. I am equally grateful for the support provided by Mr Tony Calderbank, the former Country Director of the British Council, South Sudan; Sir Richard Arden, Senior Education Advisor of DfID, South Sudan. I thank Longhorn and Mountain Top publishers in Kenya for working closely with the Ministry, the Subject Panels, UNICEF and the Curriculum Foundation UK to write the new textbooks. Finally, I thank the former Ministers of Education, Hon. Joseph Ukel Abango and Hon. Dr John Gai Nyuot Yoh, for supporting me, in my role as the Undersecretary, to lead the Technical Committee to develop and complete the consultations on the new National Curriculum Framework by 29 November 2013.

The Ministry of General Education and Instruction, Republic of South Sudan, is most grateful to all these key stakeholders for their overwhelming support to the design and development of this historic South Sudan National Curriculum. This historic reform in South Sudan's education system is intended to benefit the people of South Sudan, especially the children and youth and the future generations. It shall enhance the quality of education in the country to promote peace, justice, liberty and prosperity for all. I urge all Teachers to put this textbook to good use.

May God bless South Sudan. May He help our Teachers to inspire, educate and transform the lives of all the children and youth of South Sudan.

Deng Deng Hoc Yai, (Hon.)
Minister of General Education and Instruction, Republic of South Sudan

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Introduction

Teaching Mathematics is taking place in rapidly changing conditions. It is necessary to look for optimal didactic and educational solutions encompassing goals and contents as well as forms and teaching methods allowing for preparing students to face the challenges of the contemporary world.

The most significant role of educational system in terms of teaching Mathematics is developing and promoting subject competences as an important factor fostering student's personal development and the development of society. Well organised mathematical education facilitates logical thinking and expressing ideas, organizing own work, planning and organizing the learning process, collaboration and responsibility; it prepares for life in a modern world and enables to perform many jobs.

The teacher is required to pay more attention to students' awareness of developing learning skills and study habits, recognizing and analysing problems and predicting solutions to them. Undeniably, the implementation of modern teaching methods and techniques enhances students' curiosity about Mathematics and increases their understanding of the basis of mathematical and scientific knowledge. In accordance with the trends teaching Mathematics is supposed to help students understand and solve everyday problems.

The aim of teaching Secondary Mathematics is to encourage contemporary students to work in class, acquire knowledge and skills that are necessary in life. Moreover, research shows that teachers applying active methods assess the effectiveness of their work and how students respond to this way of teaching.

About this guide

The purpose of this guide is to offer suggestions that are helpful to Secondary 1 Mathematics teachers on planning, organizing, executing and evaluating the learning and teaching of mathematics. The suggestions will serve as useful starting points to the teachers who are expected to be dynamic innovative and creative to make the leaning process fit the learners.

The guide is to be used alongside Mathematics Students book for secondary 1. It consists of 5 units, in line with secondary 1 mathematics syllabus.

Each of the unit is structured to contain:

1. Introduction
2. Objectives
3. Teaching/Learning Activities
4. Answers to the exercises given in students secondary 1 book.

In each case, the introduction highlight the relevant work than learners are expected to have covered in their previous mathematics units and what they are expected to have covered previously. It also highlights what they are expected to cover in the unit. The teacher is expected to make a quick link up of previously learnt concepts. Learners should be able to make relevant references to their previous work. Where possible the mathematics teacher makes an entry behavior evaluation as a revision on previously learnt units related to the unit under study.

The unit objectives specify the skills (cognitive, affective, and psychomotor) that teachers will use to enable learners understand each unit. The objectives are likely to serve a useful purpose if they when stated to reflect the local conditions of the learner. For example, the type of students and the available learning resources. The teacher may break down the unit objectives to various objectives that enhance the learners understanding of the process involved and to suit different situations in the lesson, schools, society and the world at large.

Teaching/learning activities highlight the most noticeable and important. Points encountered in the learning process and suitable techniques to be used in handling each objective(s).

Answers to each exercise in the students' book are provided in these teachers guide. It is contemplated that the most conducive and favorable outcome from the guide will be realized if other sources of learning mathematics are properly organized and used.

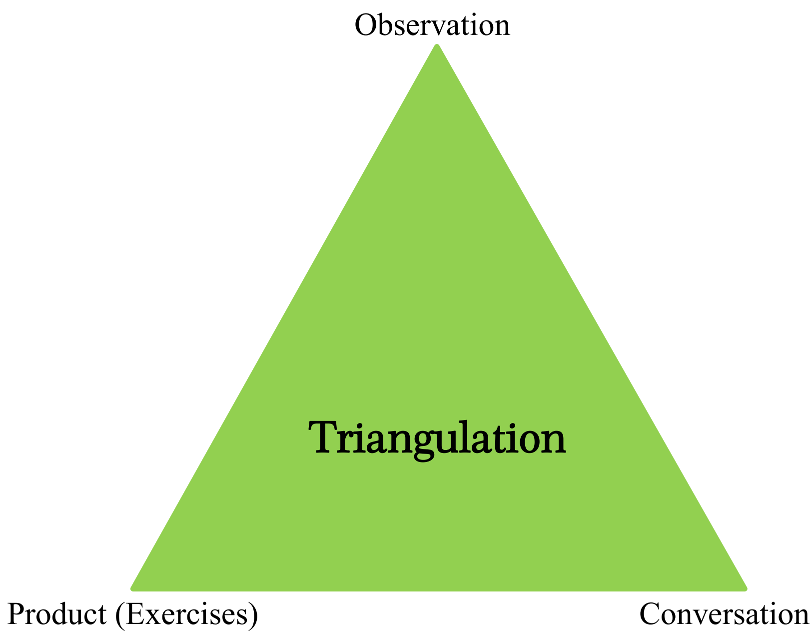
Among others, the following should be used alongside the guide:

1. The Schemes of work
2. The teacher's Lessons plans.

The Records of the work covered by the learners.

Making Classroom Assessment

• Observation – watching learners as they work to assess the skills learners are developing.
• Conversation – asking questions and talking to learners is good for assessing knowledge and understanding of the learner.
• Product – appraising the learner's work (writing report or finding, mathematics calculation, presentation, drawing diagram, etc.).



To find these opportunities, look at the "Learn About' sections of the syllabus units. These describe the learning that is expected and in doing so they set out a range of opportunities for the three forms of opportunity.

UNIT 1: NATURAL NUMBERS, FACTORS AND PROPORTION

Math: Secondary 1	**Unit 1: Natural numbers, factors and proportion**
Learn About	**Key Inquiry Questions**
Learn about: Learners should revisit fractions, percentages and decimals and explore the concepts of natural numbers such as even, odd and prime numbers. They should find the prime factors, multiples, lowest common multiple (LCM), GCD and highest common factor (HCF) of numbers, understand the characteristics and types of fractions (numerator and denominator) and simplification and how to calculate squares and square roots of numbers. Learners should work in groups to solve problems involving direct and indirect proportion and understand reciprocal of numbers and further investigate problems associated with fractions, percentages and decimals *(eg ½ = 50% = 0.5)* Learners should work in groups to investigate profit, loss, discount and commission as percentages through a personal project such as how to start a kiosk or a small business.	- What are natural numbers? - What are the types (classifications/groups) of natural numbers? E.g. prime, even and odd numbers. - How do you find squares and square roots? - How do you identify least common multiple (LCM), highest common factor (HCM,) and greatest common divisor (GCD) of any given natural numbers? - What are the types of fractions? - How do you find the reciprocal of a number? - What is profit and what is loss? - What is discount and what is commission?

Learning Outcomes

Knowledge and understanding	**Skills**	**Attitudes**
- Understand natural numbers, prime factors, multiples, GCD, HCF and LCM	- Use a number line to identify natural numbers by sign (positive/negative) and by size (order).	- Value and appreciate the importance of natural numbers in mathematics and in real life.

- Know types of fractions - Understand reciprocals of numbers - Understand profit and loss - Understand percentage discount and commission as percentage	- Use the four operations (+,-, X and /) on natural numbers. - Balance books looking at profit and loss. - Calculate squares and square roots of numbers - Solving problems involving direct and indirect proportion - Carry out an investigation about discount, loss, profit and commission.	
Contribution to the competencies Critical thinking: in solving problems Co-operation and communication: in group work		
Links to other subjects Physics, Chemistry, Biology, Geography		

Natural numbers

Natural numbers are **counting numbers**.

Exercise 1

In pairs, discuss 5 different ways numbers are useful in our lives. Write these in your exercise book and share with the class.

Expected answers

Guide the students to think of different ways numbers are important in everyday life by using questions like the ones below:

How would the world be without numbers? How useful are numbers in our everyday life? Did people use numbers before education came to Africa? How?

Encourage students to research, critically think and present the answers to these questions.

Exercise 2

In pairs discuss the following questions:

1. Is zero a counting number? Can it be used when counting?
2. What is the largest number we can count?

Encourage debate on:

i. What is the significance of zero?
ii. Is zero the same as nothing?
iii. Can we ever stop counting?

Note:

1. 0 is not a counting number
2. We can go on when counting, introduce the idea of infinity (∞ Infinity). Every learner may give different figures that they may be able to count.

Place value

Students should be made to understand significance of place value.

Explain the examples given in the student book. Let students also come up with their own examples. Check for understanding.

Factors of natural numbers

Give an example of a number like 10. Ask students to give all the numbers that can divide 10 without leaving a remainder. (These are 1, 2, 5 and 10). Explain that these numbers are called **factors.** Note that 1 and the number itself are factors.

Even and odd numbers

A natural number is even if it has 2 as one of its factors.

(Or, if it is divisible by 2)

A natural number is odd if it is not divisible by 2.

Ensure students understand the difference between odd and even numbers and how to distinguish them by asking them to give examples of even and odd large numbers. Note the characteristics: (Even numbers end with 0 or an even number while odd numbers end with an odd number.)

Students should work in groups of 4 to answer the questions in exercise 3 but write individually in their exercise books. Walk around the groups and listen to discussion. Check for understanding of place value, factors and odd & even numbers.

Exercise 3

1. Write the following numbers in numerical form;
    a. Three thousand two hundred.
    b. Twenty two thousand and fifty.
    c. One million, five hundred thousand and fifty two.
2. What is the place value of 2 in the numbers in question 1?
3. Write the largest factor other than itself of :

a. 12	b. 30	c. 44	d. 56	e. 39

4. What is the smallest whole number which?
   a. Has factors of 2, 3, 5
   b. Has factors 2, 3, 5, 7
   c. has factors 3, 5, 7

5. What kind of number do you get when you:
   a. Add three even numbers.
   b. Add three odd numbers.
   c. Multiply an even and an odd number.
   d. Add 4 consecutive odd numbers.

Expected answers

1.	a)3200	b) 22050		c) 1 500 052	
2.	a) hundreds	b) thousands & ten thousands		c) ones	
3.	a) 6	b)15	c)22	d) 28	e)13
4.	a) 30	b)210	c)105		

5. Encoruage discussion
   a) Even
   b) Odd
   c) Even
   d) Even

Prime and composite numbers

After giving definitions, ask students to give examples of both types. Ask a few to give examples on the board and have them justify why by getting the factors and proving whether the number is prime or composite.

Divide the class into groups of 4 to discuss and answer questions in Exercise 4. Each group should present their findings to the class. Check for collaboration skills, creativity and critical thinking skills amongst group members.

Exercise 4

In groups, discuss and write the answers in your exercise books

1. a. List all the prime numbers less than 50
   b. Which is the smallest prime number? What characteristic sets it apart from the others?

2. a. What is the smallest odd prime number
   b. Which is the only odd two-digit composite number less than 20?
   c. Which number is a prime number and also a factor of 105, 20 and 30.
3. The two digits of a number are the same. Their product is not a composite number. What is the original number?

Expected answers

1. a) 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

   b) 2, it's the only even prime number

2. a) 3 b) 15 c)5

3. 11

Highest Common Factor (HCF)

Use the example of the factors of 10 and 15 to explain what a common factor is. 5 can divide both 10 and 15. The highest (largest) number that is a factor of both 10 and 15 is 5. Allow students to discuss the highest common factor of

i. 14 and 28. ii. 12 and 18

HCF is also known as Greatest Common Divisor (GCD)

Exercise 5

Students should do this exercise in pairs and investigate how to find the HCF of the numbers. Check for communication skills – are they able to explain how to get the right answer? Collaboration skills- are they able to work well with their partner? Critical thinking skills - are they able to find different ways of getting the HCF?

1. Find the HCF of :
   a. 9, 12
   b. 24, 32
   c. 108, 144
   d. 25, 50, 75
   e. 22, 33, 44
   f. 10,18,20,36
   g. 32, 56, 72, 88

Expected answers

1. a) 3 b) 8 c)36 d)25
   e)11 f) 2 g)8

Lowest Common Multiple (LCM)

Explain that the multiples of 6 are the numbers that 6 can divide without leaving a remainder. These are 6,12,18,24, etc. (the multiplication tables of 6)

Group students in pairs and let them work on the investigation in Exercise 6.

Observe for communication, critical thinking and collaboration skills.

Exercise 6

In pairs, discuss and answer the following question in your exercise book.

1. List the numbers from 1 to 30.
   a. Put a circle around each multiple of 3.
   b. Put a square around each multiple of 4
   c. List the common multiples of 3 and 4 which are less than 30.
2. Find the lowest common multiple of 2, 3, 4 and 5.
3. Find the smallest natural number which is exactly divisible 2, 3, 10, 14 and 15.
4. Three bells ring at intervals of 4, 6 and 9 seconds respectively. If they all ring at 8.00 a.m. at what time will they all ring together again?

Expected answers

1. a) 12,24
2. 60
3. 210
4. LCM is 36 minutes

   So they ring together again at 8.36 am

Fractions

Activity: Use fruits like oranges, bananas.

Divide the students into groups. Each group should have some items.

1. Let each group divide 1 item among its members and note down what each number gets. Observe that this should be a **proper fraction** and highlight its meaning. Highlight the use of the words, numerator and denominator.
2. Let the group then divide 3 items among its members and discuss what each member gets. Observe that this should be a **proper fraction which can be reduced.** Check for critical and creative thinking skills. What fraction does each member get?
3. Let the group then divide 6 items among its members. Observe that their answer is an **improper fraction** and that it can also be written as a **mixed number.**

By the end of the activity, students should understand the key words: fractions 0 numerator and denominator, proper fraction, improper fraction, mixed numbers and comparing fractions (i.e $\frac{1}{2}$ *is the same as* $\frac{2}{4}$). Students should discover that to compare fractions, denominators must be the same.

Exercise 7 should be done in pairs. Check that there is transfer of concepts gained from the previous activity.

Exercise 7

1. Write each of the following in numerals:
   a. Two-thirds
   b. Three-fifths
   c. Six-sevenths
   d. Three-tenths
   e. Nine-hundredths
2. Find the value of:
   a) two-thirds of 1 hour
   b) three-fifths of 1kg
   c) one-quarter of 1 ton
3. Copy and insert the missing numbers:

   a) $\frac{2}{5} = \frac{6}{\quad}$ c) $\frac{35}{\quad} = \frac{5}{6}$

   b) c) $\frac{40}{\quad} = \frac{\quad}{15} = \frac{2}{3}$ d) $\frac{18}{90} = \frac{\quad}{30} = \frac{3}{\quad} = \frac{\quad}{5}$

4. Write three fractions equivalent to:

   a) $\frac{3}{7}$ b) $\frac{2}{5}$

5. Arrange the following fraction in ascending order;

   a) $\frac{2}{9}, \frac{4}{5}, \frac{2}{3}$ b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{10}, \frac{4}{15}$

Expected answers

1. a) $\frac{2}{3}$ b) $\frac{3}{5}$ c) $\frac{6}{7}$ d) $\frac{3}{10}$ e) $\frac{9}{100}$

2. a) 40 min b) 600g c)250kg

3. a) $\frac{2}{5} = \frac{6}{15}$ b) $\frac{35}{42} = \frac{5}{6}$ c) $\frac{40}{60} = \frac{10}{15} = \frac{2}{3}$

d) $\frac{18}{90} = \frac{6}{30} = \frac{3}{15} = \frac{1}{5}$

4. a) $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{15}{35}$ *etc*

5. a) $\frac{2}{9}, \frac{2}{3}, \frac{4}{5}$ b) $\frac{4}{15}, \frac{3}{10}, \frac{1}{3}, \frac{2}{5}$

Working with fractions: Addition and subtraction

Teacher should introduce this using an activity:

Take an orange (or any other fruit) and cut into 2. Show that the sum of the 2 parts gives 1 ($\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$)

Take 2 oranges, cut both into 4 parts. Take 1 part from the first orange and 2 parts from the second orange. What do you get? (($\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$)

Now divide 1 orange into 3 parts and the other into 4 parts. Take 1 part form the first orange and 1 part form the second orange. What fraction do you have? ($\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$)

Exercise 8

In pairs, discuss the following examples and then attempt the questions that follow. The teacher should make sure that the students show understanding by choosing 3 students to explain to the whole class.

Check that students are able to see that subtraction works the same as addition and that the most critical step is to find the LCM. Students should display analytical, co operation and communication skills in order to understand this exercise.

Evaluate a) $6\frac{3}{4}+2\frac{1}{3}$ b) $4\frac{4}{5}-2\frac{1}{4}$ c) $4\frac{1}{10}+2\frac{3}{5}-2\frac{2}{3}$

Solutions:

a) $6\frac{3}{4}+2\frac{1}{3}=8\frac{9+4}{12}=8\frac{13}{12}=9\frac{1}{12}$

Add the whole numbers
Find the LCM of 4 and 3 and add the corresponding numerators

Change the improper fraction $\frac{13}{12}$ into $1\frac{1}{12}$ and add the whole number 1 to 8.

b) $4\frac{4}{5}-2\frac{1}{4}=2\frac{16-5}{20}=2\frac{11}{20}$

Subtract the whole numbers

Find the LCM of the denominators, 5 & 4

Find the corresponding numerators and subtract.

c) $4\frac{1}{10}+2\frac{3}{5}-2\frac{2}{3}=2\frac{3+18-20}{30}=2\frac{1}{30}$

Add & subtract whole numbers: 4+2-2=2
Find LCM of 10, 5 & 3 and find corresponding numerators
Add & subtract.

Exercise 9

This exercise requires higher level thinking skills.

Group students into different ability groups (high ability students with lower ability students) and ensure that everyone understands addition and subtraction of fractions.

Negative numbers have also been introduced here. Ensure that students can add and subtract negative numbers.

Explain to each group at their point of distress.

Evaluate and simplify:

1. $6\frac{1}{2} + 3\frac{3}{4}$
2. $2\frac{7}{8} + 1\frac{4}{5}$
3. $4\frac{1}{5} + 2\frac{3}{10}$
4. $5\frac{3}{4} - 2\frac{7}{8}$
5. $3\frac{1}{2} - 2\frac{3}{4}$
6. $10\frac{7}{16} - 3\frac{3}{8}$
7. $-\frac{2}{3} - \frac{4}{5} + \frac{3}{10}$
8. $3\frac{1}{6} - 2\frac{1}{3} + \frac{7}{12}$
9. $-2\frac{6}{13} + 1\frac{1}{2}$
10. $4\frac{1}{6} + 1\frac{11}{20} - 3\frac{7}{15}$
11. $10\frac{1}{2} - 3\frac{2}{3} + 4\frac{1}{5} + 2\frac{3}{4}$

Expected answers

1. $10\frac{1}{4}$
2. $4\frac{27}{40}$
3. $5\frac{1}{2}$
4. $2\frac{7}{8}$
5. $\frac{3}{4}$
6. $7\frac{1}{16}$
7. $-1\frac{1}{6}$
8. $1\frac{5}{12}$
9. $-1\frac{1}{6}$
10. $2\frac{1}{4}$
11. $\frac{-25}{26}$

Working with fractions: Multiplication

Use the examples to explain the principal of multiplication of fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Divide the class into groups and allow them to discuss the questions in the following exercise. Ensure that their knowledge of negative numbers is applied correctly. Give examples like $-2 \times 4 = -8$ just incase.

Check for communication skills – the answer should be written correctly, neatly and simplified completely, collaboration skills – each member of the group should show understanding, analytical skills – each member of the group should each step to take to arrive at the answer.

Exercise 10

Evaluate the following showing all your working clearly and simplifying where possible

1. Find:

a. $\frac{2}{5} \times \frac{3}{4}$ b. $-\frac{5}{4} \times \frac{2}{3}$ c. $\frac{4}{27} \times \frac{21}{16}$

2. Find:

a. $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$ b. $\frac{3}{8} \times \left(-\frac{2}{3}\right) \times \frac{3}{5}$ c. $\frac{2}{5} \times 4 \times 1\frac{5}{8}$

d. $\frac{2}{3} + \frac{3}{4} \times \frac{2}{3}$ e. $\frac{3}{5} - \frac{1}{3} \times \frac{6}{7}$ f. $\frac{2}{3} + \frac{3}{4} \times \frac{2}{3} - \frac{1}{2}$

Expected answers

1 a) $\frac{3}{10}$ b) $-\frac{5}{6}$ c) $\frac{7}{36}$ d) $\frac{1}{2}$

2 a) $\frac{1}{10}$ b) $-\frac{3}{20}$ c) $2\frac{3}{5}$ d) $1\frac{1}{6}$

e) $\frac{11}{35}$ f) $\frac{2}{3}$

Reciprocals

Task

See if you can solve these problems in your head by multiplying the first number by the reciprocal of the second number:

1. $8 \div \frac{1}{5}$
2. $10 \div \frac{1}{10}$
3. $3 \div \frac{3}{8}$

(*Hint:* All the answers will be whole numbers.)

Working with fractions: Division

Activity:

1. In groups of 4, let students divide an orange into 4. What fraction is each part? Link that to $1 \div 4 = \frac{1}{4}$
2. Take half an orange and divide it into 2. What fraction do we get? Link that to $\frac{1}{2} \div 2 = \frac{1}{4}$. This is the same as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. This is the same as multiplying the first fraction with the reciprocal of the second fraction.
3. Ask the students to think of other divisions using the orange and prove their answer using the analytical method (calculation)

Without using oranges, let students discuss the questions in Exercise 11. They should discover that just like in multiplication, you have to convert a mixed number to an improper fraction in order to solve the question.

Check for understanding of the concept, critical thinking skills and correct steps written.

How to Make a Fraction on a Scientific Calculator

By default, scientific calculators, like regular ones, display fractions as decimals. So if you enter a simple fraction, such as $\frac{1}{2}$, the display reads 0.5. Some – but not all – scientific calculators offer a feature that allows you to display fractions without making the conversion. Using this feature, you can enter a complex fraction and simplify it right on your calculator. Calculators with this feature also allow you to enter a number composed of an integer and a fraction, such as $1\frac{1}{4}$. If your calculator doesn't have this feature, you can use a workaround to manipulate fractions.

The Fraction Button

Calculators that display fractions sometimes have a special mode, called Math mode that you must first select before you can enter fractions. When the calculator is in Math mode, the word "math" appears at the top of the screen. Once you have selected this mode (if necessary), look for a button with two boxes, one black and one white, arranged on top of each other with a horizontal line between them. This is the fraction button. On some models, the button may show x/y or a b/c. Pressing this button enables the fraction feature.

Entering a Fraction

Enter the Numerator

When you press the fraction button, a fraction template appears in the display. It sometimes consists of two blank boxes arranged one over the other and separated by a horizontal line. The cursor will appear in the top box. You can now enter the numerator of the fraction.

On some models, fractions appear as numbers separated by an inverted "L." This character represents the horizontal line that separates the numerator and denominator.

Enter the Denominator

Press the cursor down key (the key with the arrow that points downward) to move the cursor from the top box in the display to the bottom if you calculator has number boxes. You can now enter the denominator. If you need to change the numerator, you can always return to the top box by pressing the cursor up key.

If you have the type of calculator that shows fractions in a single line, simply enter the denominator. There's no need to move the cursor.

Use the Shift Key to Enter a Mixed Number

If you want to enter a number such as 1 1/4, press the shift key before pressing the fraction key. The display will show a third box to the left of the two fraction boxes, and the cursor will be in that box. Enter the integer part of the number, then press the cursor right key to move the cursor to the numerator box of the fraction.

On calculators with linear displays, enter the three numbers in this order: integer, numerator, and denominator.

Handling Fractions on Calculators without the Fraction Key

Although you can't display non-decimal fractions on a calculator without a fraction function, you can still enter them. First enter the numerator of the fraction, then press the division key and enter the denominator. Hit the "equals" key and the fraction will display as a decimal.

You can't convert a decimal to a fraction on the calculator, but the calculator can help you do it with a pencil and paper. Suppose you want to express 0.7143 as a fraction. You could write it as $\frac{7143}{10\,000}$, but maybe you want to reduce this to something a lot simpler, such as a denominator that's a single digit. To do this, enter the original number as decimal, and then multiply by the desired denominator. This gives you the numerator of the fraction. For example, if you want a fraction with 7 in the denominator, multiply 0.7143 by 7. The calculator will display the numerator, which in this case is 5.0001, which is close enough to 5 to be equal. You can then write the fraction $\frac{5}{7}$ on a piece of paper.

Exercise 11

In pairs, discuss how you would divide the following fractions

a. $1\frac{1}{3}, 3\frac{1}{2}$

b. $2\frac{3}{4}, \frac{2}{3}$

c. $\frac{4}{5}, 3$

d. $\dfrac{2+\frac{1}{3}}{2-\frac{1}{3}}$

e. $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}-\frac{3}{4}}$

f. $\dfrac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{5}}$

Expected answers

a) $\frac{8}{21}$

b) $4\frac{1}{8}$

c) $\frac{4}{15}$

d) $1\frac{2}{5}$

e) -17

f) $\frac{35}{48}$

Fractions, percentages and decimals

Have students give you their understanding of a decimal.

There are many methods to change fractions to decimals. Check for understanding by asking students to clearly show how to change fractions to decimals: e.g. i. $\frac{1}{2}$ ii. $\frac{3}{4}$ iii. $\frac{7}{10}$

Students should discuss how to change decimals to fractions. Use questions to check for understanding: e.g. $0.25 = \frac{1}{4}$, $0.9 = \frac{9}{10}$ etc

Recurring decimals: Explain to the class the steps needed to change a recurring decimal to a fraction

Exercise 12

1. In a closing-down sale, a shop offers 50% off the original prices. What fraction is taken off the prices?

   Correct answer: $50\% = \frac{50}{100} = \frac{1}{2}$

2. In a survey, one in five people said they preferred a particular brand of milk. What is this figure as a percentage?

   Correct answer: one in five is the same as saying $\frac{1}{5}$ of the people preferred a particular brand of milk. So $(1 \div 5) \times 100 = 20\%$

3. Keji is working out a problem involving $\frac{1}{4}$. She needs to enter this into a calculator. How would she enter $\frac{1}{4}$ as a decimal on the calculator?

   Correct answer: $\frac{1}{4} = 1 \div 4 = 0.25$

4. Mr. Achan pays tax at the rate of 25% on his income. What fraction is Mr. Achan's income in this?

   Correct answer: $25\% = \frac{25}{100} = \frac{1}{4}$

5. When Deng was buying his plot of land he had to put down a deposit of $\frac{1}{10}$ of the value of the piece of land. What percentage was this?

   Correct answer: $\frac{1}{10} = (1 \div 100) \times 100\% = 10\%$

6. I bought a coat in the December sales with $\frac{1}{3}$ off the original price. What percentage was taken off the price of the coat?

   Correct answer: $\frac{1}{3} = (1 \div 3) \times 100 = 33.33$. This is 33% to the nearest percentage.

7. Mary bought some fabric that was 1.75 metres long. How could this be written as a fraction?

Correct answer: $0.75 = \frac{75}{100} = \frac{3}{4}$. So 1.75 metres written as a fraction is $1\frac{3}{4}$ metres.

Standard form

Standard Form (or Standard Index Form) is a way of writing very big or very small numbers in a nice short way by using powers of 10. This is because when you multiply by 10, 100 (10^2) or 1000 (10^3) and so on the effect is that the decimal point moves.

To be technical about it, every number can be written as $\mathbf{a \times 10^n}$ where **a** is a number between 1 and 10 and **n** is an integer (whole number including negative whole numbers)!

For example:
6 000 000 000 can be written as $\mathbf{6 \times 10^9}$
0.32 can be written as $\mathbf{3.2 \times 10^{-1}}$ (Note the **negative** power of 10 because you want the point to move to the **left** and not to the right).

What you need to do:

1. Look at your number written in full and put the decimal point straight after the first non-zero digit.
2. Count how many times it needs to shift to get back to where it was. This gives you your power of 10.

Take this example...
To write 5480000 in standard form, shift the decimal point until it is after the 5, counting how numbers it hopped over on the way. That number is then the number you have to raise 10 to to give you the standard form. **Click 'Play' below to see this in action.**
Now you can put numbers into your calculator that wouldn't even fit on the screen before!

Calculating with standard form

As numbers written in Standard Form are still numbers you can multiply them, divide them, etc. The rules of indices can provide some shortcuts!

For example:

$(2 \times 10^7) \times (4 \times 10^9)$ can be rearranged to $(2 \times 4) \times (10^7 \times 10^9)$ giving 8×10^{16}

Important: **The number at the front must be between 1 and 10 so be careful! You may need to adjust it a bit by moving the point and changing the power of 10 to make up for your change.**

For example:

$(5 \times 10^6) \times (7 \times 10^{12}) = (5 \times 7) \times (10^6 \times 10^{12}) = 35 \times 10^{18}$

But the answer is not in Standard Form and the question may ask you to give your answer in Standard Form

Shift the point over the 5 to get 3.5 then increase the power of 10 by **one** to get it back where it was before you moved it. This gives the answer $\mathbf{3.5 \times 10^{19}}$.

Exercise 13

In pairs, convert these numbers into standard form.

1. 0.000000000467 – 4.67×10^{-5}
2. 0.0000000316 - 3.16×10^{-8}
3. 547 - 5.47×10^{2}
4. 0.00000000509 - 5.09×10^{-9}
5. 4660000000 - 4.66×10^{9}
6. 11000000 - 1.1×10^{7}
7. 0.000703 - 7.03×10^{-4}
8. 2490000 - 2.49×10^{6}
9. 0.0768 - 7.68×10^{-2}
10. 54400 - 5.44×10^{4}

Percentages

We use percentages to compare a portion with a whole amount of 100%.

Exercise 14

1. **Write these decimals as fractions:**

   a) 0.3
   b) 0.5
   c) 0.6
   d) 0.02
   e) 0.05
   f) 0.25
   g) 0.36
   h) 0.125

2. **Write these fractions as decimals:**

   a) $\frac{7}{10}$
   b) $\frac{1}{5}$
   c) $\frac{2}{5}$
   d) $\frac{3}{4}$
   e) $\frac{7}{8}$
   f) $\frac{2}{3}$
   g) $\frac{9}{20}$
   h) $\frac{7}{25}$

3. **Write these percentages as decimals:**

   a) 3%
   b) 30%
   c) 25%
   d) 80%
   e) 8%
   f) 12%
   g) 67%
   h) 17.5%

4. **Write these percentages as fractions:**

   a) 20%
   b) 75%
   c) 5%
   d) 30%
   e) 40%
   f) 15%
   g) 24%
   h) 35%

5. Write these decimals as percentages:

a) 0.25
b) 0.5
c) 0.7
d) 0.07
e) 0.45
f) 0.09
g) 0.4
h) 0.375

6. Write these fractions as percentages:

a) $\frac{1}{10}$
b) $\frac{1}{5}$
c) $\frac{9}{10}$
d) $\frac{3}{4}$
e) $\frac{4}{5}$
f) $\frac{17}{20}$
g) $\frac{1}{3}$
h) $\frac{2}{3}$

At the end of the exercise, have the students:

1. Explain how to convert...
   - a decimal to a fraction
   - a fraction to a decimal
   - a percentage to a fraction or decimal
   - a fraction or decimal to a percentage.

2. Give some examples of real contexts where...
   - decimals are used
   - fractions are used
   - percentages are used.

Profit and Loss

Definitions

Cost Price: The price (amount) paid to purchase a product or the cost incurred in manufacturing a product is known as the cost price of that product.

Types of cost

1. **Fixed cost:** It is a type of cost which is fixed under all conditions and does not vary according to the number of units produced.
2. **Variable cost**: Variable cost is a type of cost which varies according to the number of units.
3. **Semi-variable cost:** As the name suggests, these costs are the ones that are fixed in part and variable in part. Effectively, this is the case that we see most often. Imagine the scenario in a factory. There is a capital cost, which remains the same under all conditions (fixed cost) and a variable cost of the product, which in turn depends upon various factors.

Selling Price: The price at which a product is sold is called the selling price of the product.

Marked Price: Do shopkeepers put up price on the label that they wish to sell on or they put up an inflated price? If you think closely, majority of the shopkeepers mark-up their products, in anticipation of the discounts they would have to offer. This is a clever way of operating. Mark-up the price in advance, offer a discount and make the customer feel happy, and then sell the product.

List Price: List price or the tag price is the price that is printed on the tag of the article. For all practical purposes, we assume it to be same as the marked-price.

Margin: The profit percentage on selling price is known as MARGIN.

Profit and Loss Basic Concepts

1. One can generate a profit only if Selling Price> Cost Price
2. One generates a loss when Selling Price < Cost Price.
3. Profit = Selling Price – Cost Price
   % Profit = ({(Selling Price -Cost Price)/Cost Price} x 100)
4. Loss = Cost Price – Selling Price
   % Loss = ({(Cost Price-Selling Price)/Cost Price} x 100)

Exercise 14

Check that the students show the steps clearly and show understanding of profit and loss.

1. A trader bought 8 trays of eggs at SSP 240 per tray, eight eggs broke and he sold the rest at SSP 12 per egg. If a tray holds 30 eggs, how much money did he get as profit?

Answer: SSP 864

2. Jacob bought 250 chicken whose average mass was $1\frac{1}{2}$kg. The buying price per kilogram was SSP 150. He then sold each chicken for SSP 300, what profit did Jacob make?
   Answer: SSP 18 750

3. Jacinta bought 15 bags of fruits at SSP 450 per bag. She spent SSP 500 on transport, $1\frac{1}{2}$ bags of the fruits got spoilt and she sold the rest at SSP 400 per bag. What was her loss?
   Answer: SSP 1850

4. Joel bought 5 pairs of shoes at SSP 250 per pair. He later sold them. If the buyer gave six two hundred Sudanese pound notes and Joel gave him back SSP 105. How much was the loss?
   Answer: SSP 155

Exercise 15

This exercise requires higher level thinking and should be done as an assignment. Students should use their knowledge of numbers to answer the questions. Students should discover the easiest way to find the solution and show their working clearly. They should present the work to the teacher for marking.

In groups, discuss the following questions.

1. The price of milk is raised from SSP 50 to SSP 55 per litre.
   a. Juma buys 5 litres every day. How much more money is he spending every day because of this price increase?
   b. Mary has a shop and buys milk to sell. How much was she spending before the price increase if she buys 10 litres every day?
   c. She sells her milk at SSP 55 per litre, what was her percentage profit?
   d. After the price increase, she decides to sell her milk at SSP 65. What is her percentage profit now?

Expected answers

1. a) SSP 25
   b) SSP 500
   c) 10%
   d) 18.2%

Discount

Discounts are reductions in the selling price of an article or the amount of a bill. Discounts are expressed as rates or percent. The list price of an article is the original price. The net price is the list price minus the discount. To find the amount of a discount, multiply the list price, or base, by the rate of discount.

Formula: $P = B \times R$

Example

A used car listed at $925.00 was sold at a discount of 10%. Find the discount.

$$B = SSP\ 925,$$

$$R = 10\%\ P = ?$$
$$P = B \times R;$$

$$P = SSP\ 925\ x\ .10$$

$$= SSP\ 92.50$$

To find the net price, subtract the amount of the discount from the original or list price.

Formula: D = B - P.

Example

A file cabinet was listed at SSP 62.50. Find the net price if the discount was 12%.

$$D = ?\ P = ?\ R = 12\%\ B = SSP\ 62.50.$$
$$P = B \times R$$

$$= SSP\ 62.50 \times .12.\ \ B$$

$$= SSP\ 7.50.$$

$$D = B - P$$

$$= SSP\ 62.50 - SSP7.50$$

$$= SSP\ 55.00\ net.$$

Alternate Solution
Instead of multiplying the list price by the percent discount and deducting the result, a short method is to deduct the percent discount from 100% and multiply the remainder by the list price to get the net price directly. Thus in the same example we have:

$$100\% - 12\% = 88\%;$$
$$62.50 \times .88 = SSP\ 55.00$$

Commission

Commission is the amount of money paid to an agent for buying or selling goods. When the commission is some percent of the value of the services performed, that rate percent is called the rate of commission.

Gross proceeds or selling price is the money received by the agent for his employer. In commission calculations this is called the base. Net proceeds is the gross proceeds minus the commission.

To find the commission, multiply the principle amount, or base, by the rate of commission.

Formula: $Commission\ (P) = Base\ (B) \times R\ (R),$
or... $P = B \times R$

Example
A real estate agent sold a piece of property for SSP 50 000. His commission is 7.5%. How much does he receive?

P = SSP 50 000,   B = 7.5%   P = ?

$SSP\ 50\ 000\ \times\ .075\ =\ SSP3750.00\ commission$

Example

A salesman received SSP 630 for selling a boat. His rate of commission was 5.25%. What was the selling price?

$$\frac{P}{R} = B$$

$$P\ =\ 630;\quad R\ =\ 5.25,\ \ R\ = ?$$

$$SSP\ 630\ /\ .0525\ =\ SSP\ 12\ 000\ selling\ price$$

Project work (to be done in groups)

Divide students into groups of 4

Give each group the task of operating a small canteen with a starting capital of SSP 1000.

Each group should come up with a list of different items they can stock their canteen with.

They should determine the SP of each item and sell to other members of the class. After a period of time (1 week) the group should list items left, determine profits or losses for each item and overall profit and loss of their canteen.

Each group will present to the class clearly showing

a) a list of items bought and item sold over the period of time
b) the profit(loss) made on each item and the overall profit of the canteen
c) percentage profit(loss) on each item and the overall percentage profit(loss)
d) how to get the SP of an item if you want to make 10% interest on that item
e) the strategies that one can take to increase profits or minimize losses.

Grade them on how the presentation of their accounts, creativity on what they decided to stock their canteen with, the most profitable canteen, the group that shows the best collaboration skills and that shows understanding of fractions, decimals, percentages and profits and losses

Ensure understanding of the terms defined in the students' book.

UNIT 2: MEASUREMENT

Math: Secondary 1	**Unit 2: Measurement**
Learn About	**Key Inquiry Questions**
Learners should work in pairs or groups to first estimate and then calculate and measure the length, area, volume and capacity of a range of given shapes and objects using metric notation. They should work in groups to design and carry out an investigation to find if there is a relationship between the surface area and volume of a solid object. They should measure the surface area and volume of cylinders and derive the formulae that enable these to be calculated. They should apply this to other solid objects. They should explore the differences between volume and capacity (content) of a container and apply the knowledge to investigate irregular surfaces and volumes. They should be given problems to solve that involve the use of standard measure. Where possible, learners should use computer software to draw three-dimensional solids (3D).	• What are the standard of length, area, volume and capacity? • How do we find a formula to calculate the surface area and volume of cylindrical solids? • Is there a relationship between surface area and volume?

Learning Outcomes

Knowledge and understanding	**Skills**	**Attitudes**
• Understand the use of standard metric measures.	• Measure and calculate length, area, volume and capacity. • Plan and carry out an investigation to determine if there is a relationship between surface area and volume	• Appreciate the importance of accurate measurements.

	- Derive the formulae that allow volume and capacity to be derived. - Solve problems involving length, area, volume and capacity - Use computer technology, where possible, to draw 3D solids.	
Contribution to the competencies Critical thinking: in solving problems Co-operation and communication: in group work		
Links to other subjects Physics, Geography		

The teacher should guide students on discussion on measurements and how they are applied in real life.

The students should be able to estimate distances and us the correct unit to make these estimates.

Students should be aware on how to convert from different units in the metric system. For instance, from km to cm and vice versa

Discussion question: Let students give reasons for their answers. Pick a few to present to the class (presentation skills)

Total length of rope =72 m

Total length of posts = 6m

Area inside ring = 36 m^2

Exercise 1

Divide students in groups, guide discussion and let the groups present to the whole class: ask for reasons why they think the unit of length should be used.

Conversion of units

Students can convert within the metric measurement units for length, area, capacity, mass and time, and between units of mass, capacity and volume.

Initially, the metric system was designed so that 1 gram was defined as the mass of 1 cubic centimetre of pure water at 4 degrees Celsius and the litre was defined as the volume of 1 cubic decimetre (10cm × 10 cm × 10 cm). This allows conversion between these units.

Success depends on assembling a variety of pre-requisite knowledge:

- knowledge of metric units and metric prefixes (milli, Mega etc.)
- knowledge of decimal notation

- ability to multiply and divide by powers of ten, including with decimals
- elementary proportional reasoning
- knowledge of approximate size and common uses of metric units.

Exercise 2

1 a) 800 cm b) 16400 cm c) 4 cm d) 600 000 cm e) 500 000 cm

2 a) 5000 m b) 2 000 000 m c) 0.6 m d) 36.5 m e) 6 m

Perimeter

Exercise 3

Students can work in pairs. Units should be written with the correct figure. Where the units in the question are different, students should convert to one unit then add to get perimeter.

1. Ensure that they can use the ruler correctly to measure the lengths.

2. a) 15 cm b) 21 m c) 120 cm d) 12 cm/ 120 mm

Circumference of a circle

Activity 1

Divide the class in groups of 4/5
Check for creativity (the kind of containers they get), following instructions, collaboration skills, critically thinking and organizational skills. Their work should be neat and they should finally come up with an approximation for π.

Exercise 4

1. a) 37.704 cm b)16.34 cm

2. 15.71 m

3. a) 251.36 cm b) 25.136 km c) 3979 times

4. a) 25.71 cm b) 23.43 cm

5. It's the same distance: 12.57 m

Area

Activity 2

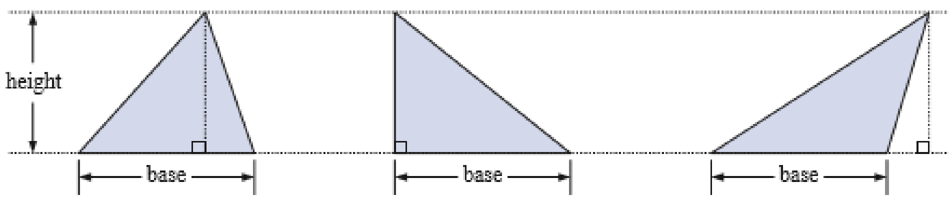
Divide students in groups of 4 and give squared paper to each group

1. Students should draw different sizes of the following polygons on squared paper
    - i) Rectangle
    - ii) Triangle
    - iii) Parallelogram
    - iv) Trapezium
2. Let them count the squares inside each shape and estimate its area
3. Using the fact that $Area\ of\ a\ rectangle = length \times width$ cut out the other shapes to form a rectangle and calculate the areas of these polygons.
4. Come up with a general formula for calculating the area of these shapes.

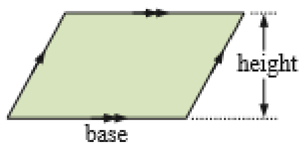
Check for creativity, critical thinking, collaboration and analytical thinking skills.

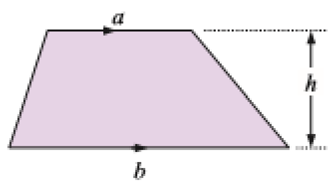
They should finally come up with the formulas below:

- **Area of a triangle** = $\frac{1}{2} \times base \times height$



- **Area of a parallelogram = base x height**





- **Area of a trapezium** = $\frac{(a+b)}{2} \times h$

Task 3: Length and Area

In groups,

- Estimate the length and width of the floor of your class room. Give reasons for your answer.
- Explain how you would measure the length and width of your classroom.
- Measure and give your measurements in cm and m.
- Hence find its area in cm^2 and m^2.
- How do we convert from cm^2 to m^2 and from m^2 to cm^2?

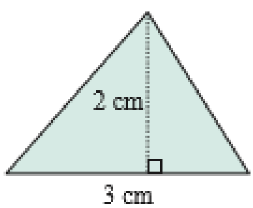
The students should first estimate the length and width of the class room then find ways of measuring. Encourage different ideas. Students should know that there is more than one way to get the correct answer.

Students should then use the formulas to answer the questions in exercise 5. They should work in pairs so as to promote collaboration and cooperation skills. Encourage discussion and communication.

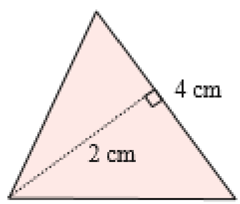
Exercise 5

. Find the areas of the shapes.

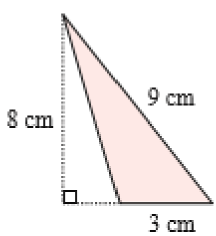
a)



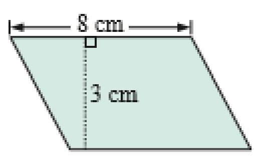
b)



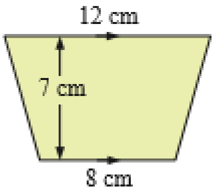
c)



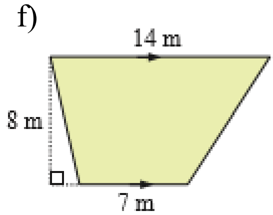
d)



e)



f)



Expected answers

a. 3 cm^2
b. 4 cm^2
c. 12 cm^2
d. 24 cm^2
e. 70 cm^2
f. 84 cm^2

Area of a circle: Use π=3.142

Activity

In the same groups as before, students should draw different sizes of circles on squared paper.

Let students use the squares to estimate the area and then find a analytical way to find the area of a circle using radius and π. Encourage creativity, there are different ways to get the area: **π (Pi)** times the Radius squared: $A = \pi r^2$

or, when you know the Diameter: $A = (\pi/4) \times D^2$

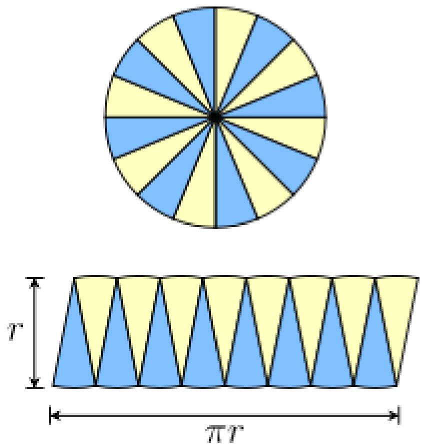
or, when you know the Circumference: $A = C^2/4\pi$

How does the area of a circle relate to the area of a parallelogram?

Answer:
If you divide a circle into a number of equal segments then stack them in a row head-to-tail, the resulting shape is like a parallelogram with bumpy sides, with the same area as the circle.

Explanation:
As you make the segments smaller and smaller, the parallelogram becomes more of a rectangle with shorter side equal to the radius of the circle r and longer side πr - half of the circumference of the circle.

Hence we get the formula πr^2 for the area of a circle of radius r.

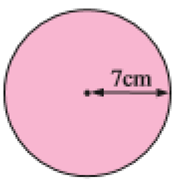


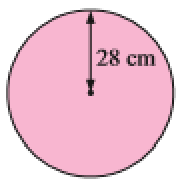
After the activity, go over the examples with class. Let student discover whether their formulas work.

Exercise 6: To be done in pairs

Check for understanding and correct working.

1. Find the area of the following, giving your answer to 2 decimal places

a)


b)


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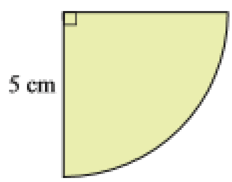
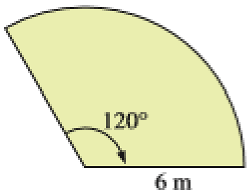
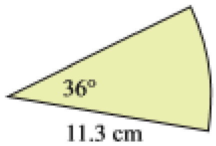


d)

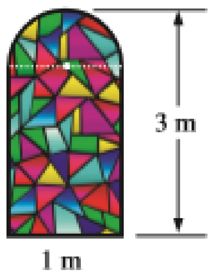


e)



2. A circle has a diameter of 2.6 cm. Find, correct to 2 decimal places
   a) its perimeter b) its area

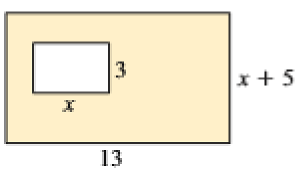
3. A goat is tied to a post by a rope 5.4 m long. What maximum area can the goat graze?
4. The illustration shows the dimensions of a stained glass window. Find the area of the window



5. A rectangle is 12cm by 8 cm. If the length of the rectangle is increased by 4cm, by how much must the width be changed so that the area remains the same?
6. Find the area of the shaded region in the figures below(the measurements are in cm)

7.

a)



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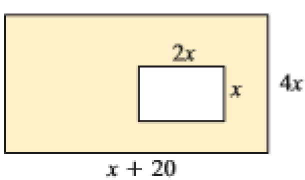

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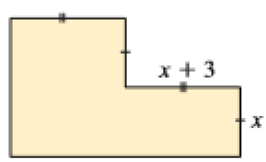


Expected answers

a) 153.96 cm^2

b) 2463.33 cm^2

c) 19.64 cm^2

d) 37.70 cm^2

e) 40.12 cm^2

2. a) 8.17 cm b) 5.31 cm^2

3. 91.62 m^2

4. 2.89 m^2

5. reduced by 2 cm

6. a) $10x + 65cm$ b) $(2x^2 + 80x)cm$ c) $(3x^2 + 9x)cm$

Surface Area:

Task 3

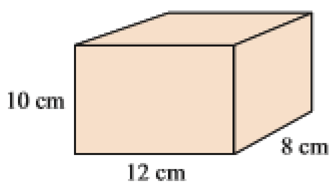
Students to work in groups to find the surface area of the classroom in metres. Ensure they are able to convert from cm^2 or km^2?

Exercise 7

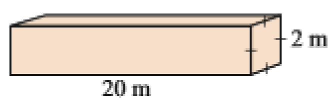
1. Find the total surface area of a cube with sides 3 cm

2. Find the surface area of the following rectangular prisms:

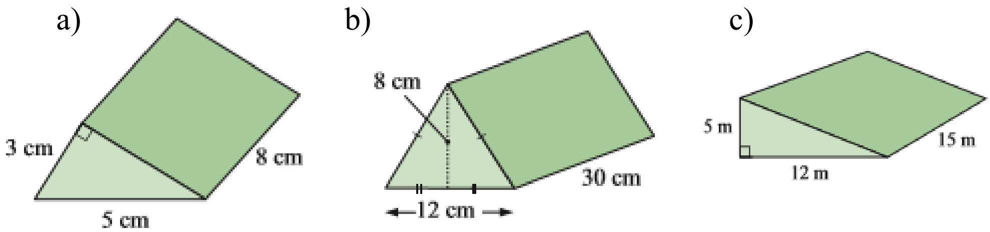
a)



b)

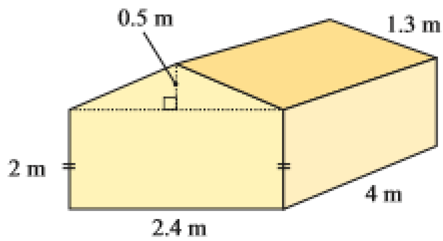


3. Find the surface area of the following triangular prisms:



4. A room has dimensions 4 m by 3 m by 2.4 m high. Find the cost of painting the inside of the room (walls & ceiling) if 1 litre of paint costs SSP13.20 and each litre covers 5 m^2.

5. A tent made from canvas is shown below. Find the total cost of the canvas if it costs SSP12.80 per square metre. (the tents floor is also made from canvas)



Expected answers

1. 24 cm^2

2. a) 592cm^2 b) 168 m^2

3. a) 108 cm^2 b) 1056 cm^2 c) 510m^2

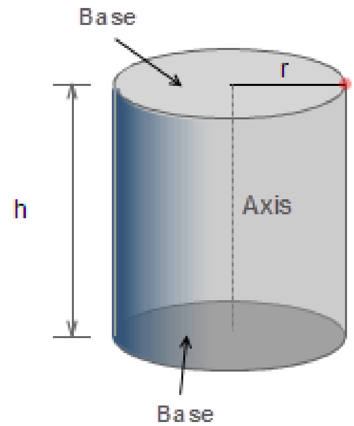
4. 45.6m2 therefore 10 cans needed and cost is SSP 132

5. SSP 617

Cylinders & spheres

Cylinders: Base and side

A cylinder is a geometric solid that is very common in everyday life, such as a soup can. If you take it apart you find it has two ends, called bases that are usually circular. The bases are always congruent and parallel to each other. If you were to 'unroll' the cylinder you would find the side is actually a rectangle when flattened out.



Height

The height ***h*** is the perpendicular distance between the bases. It is important to use the perpendicular height (or 'altitude') when calculating the volume of an oblique cylinder.

Radius

The radius ***r*** of a cylinder is the radius of a base. If you are given the diameter instead, remember to halve it.

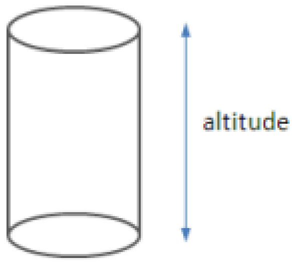
Axis

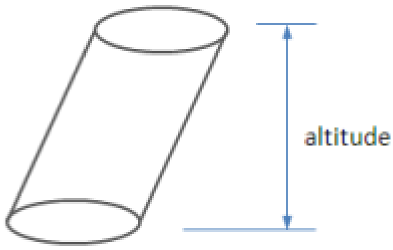
A line joining the center of each base.

Right and oblique cylinders

When the two bases are exactly over each other and the axis is a right angles to the base, this is a called a 'right cylinder'. If one base is displaced sideways, the axis is not at right angles to the bases and the result is called an oblique cylinder. The bases, although not directly over each other, are still parallel.

In the applet at the top of the page, check the "allow oblique" box and drag the orange dot sideways to see an oblique cylinder.





Right Cylinder Oblique Cylinder

Surface area

The surface area of a cylinder can be found by breaking it down into three parts:

- The two circles that make up the ends of the cylinder.
- The side of the cylinder, which when "unrolled" is a rectangle

- The area of each end disk can be found from the radius ***r*** of the circle. The area of a circle is πr^2, so the combined area of the two disks is twice that, or $2\pi r^2$.

- The area of a rectangle is the width times height. The width is the height $\boldsymbol{h}$ of the cylinder, and the length is the distance around the end circles. This is the circumference of the circle and is $2\pi r$. Thus the rectangle's area is $2\pi r \times h$.

 Combining these parts we get the final formula:

$$area = 2\pi r^2 + 2\pi rh$$

where:
π is Pi, approximately 3.142

r is the radius of the cylinder
h height of the cylinder

By factoring $2\pi r$ from each term we can simplify this to,

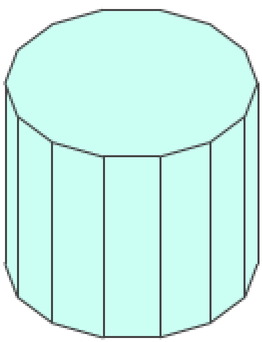
$$area = 2\pi r(r + h)$$

The shape of the bases

Usually the bases are circles, so a familiar soup can would be technically called a 'right circular cylinder'. This is the most common kind, and if someone just says 'cylinder' this is usually what they mean. The bases can however be almost any curved shape, but the most common alternative to a circle is an ellipse. The shape would then be called an 'elliptical cylinder'.

Relation to a prism

A prism is a solid with bases that are polygons and the sides are flat surfaces. Strictly speaking a cylinder is not a prism, however it is extremely similar. In a prism where the bases are regular polygons, the prism begins to approach being a cylinder when the number of sides is large.



Sphere

The surface area of a sphere is given by the formula

$$Area = 4\pi r^2$$

where r is the radius of the sphere.

This formula was discovered over two thousand years ago by the Greek philosopher Archemedes. He also realized that the surface area of a sphere is exactly equal to the area of the curved wall of its circumscribed cylinder, which is the smallest cylinder that can contain the sphere.

If you know the surface area

By rearranging the above formula you can find the radius:

$$r = \sqrt{\frac{a}{4\pi}}$$

where a is the surface area.

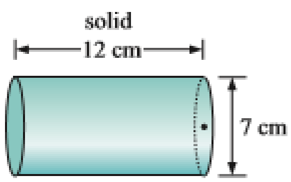
Interesting facts

1. For a given volume, the sphere is the shape that has the smallest surface area. This why it appears in nature so much, such as water drops, bubbles and planets.
2. The surface area of a sphere is exactly four times the area of a circle with the same radius. You can see this in the area formula, since the area of a circle is πr^2 and the surface area of a sphere is $4\pi r^2$

Exercise 8

1. Find the surface area of the following shapes:

a)



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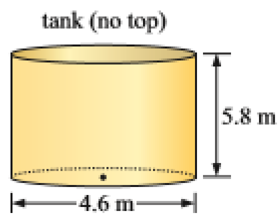
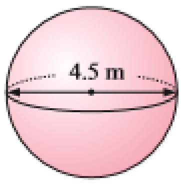


c)



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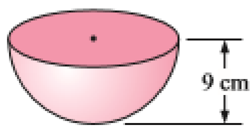


2. Using the formula for the surface area of sphere, find the surface area of a soccer ball of diameter 20 cm.

3. An 8 cm by 10 cm rectangle has the same perimeter as an isosceles triangle with base 10 cm and equal sides 13 cm. Which figure has the greatest area and by how much?
4. Determine how much paint is required to paint the outside of a cylindrical tank 12 m long with a diameter 10 m if each litre of paint covers $15m^{2}$.

Expected answers

1. a) $341cm^2$ b) $100m^2$ c) $11300cm^2$ d) $509cm^2$

2. $1260cm^2$

3. Rectangle by $20cm^2$ extra

4. 35.6 litres

Volume

Guide the students on a discussion on the amount of water the glasses contain, is it the same? If not, why?

Students should understand how to convert units of volume. When a solid is given with different units of measurements, it is easier to convert to one unit before getting the volume.

Students should understand the volume formulae and what the cross section is. Use the example of a loaf of bread to explain the cross section.

Investigation

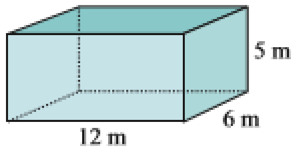
Divide the students into groups, each group should perform the task and compare their results at the end of the investigation.

Down load the printable nets of the solids or let the students draw the nets on Manila paper and cut out to form the solids.

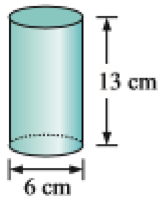
Exercise 9

Find the volume of the following:

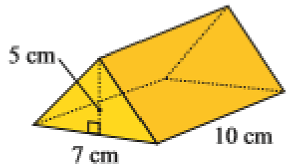
1. a)



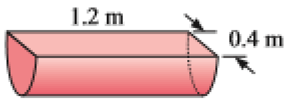
b)



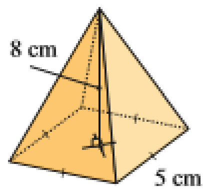
c)



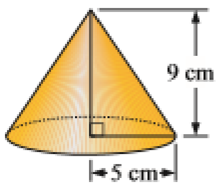
d)



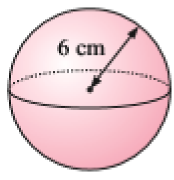
e)



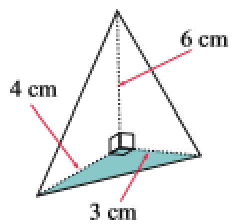
f)



g)

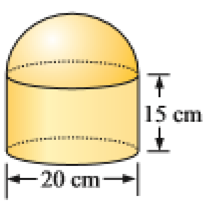


2. A triangular based pyramid is shown.
   Find its volume.



3. Find the volume of the figures below:

a)



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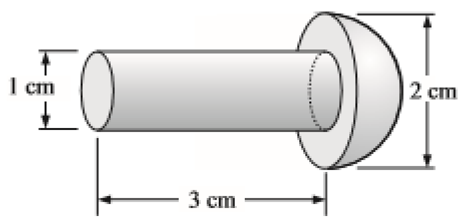


Expected answers

1. a) $360m^3$
   b)$368\ cm^3$
   c)$175\ cm^3$
   d) $0.0754\ m^3$
   e)$66.7\ cm^3$
   f) $236\ cm^3$
   g) $905\ cm^3$

2. $12\ cm^3$
3. a) $6810\ cm^3$ b) $4.45\ cm^3$

Capacity

Task: Students should work in pairs.

Ensure each group has containers of different shapes.

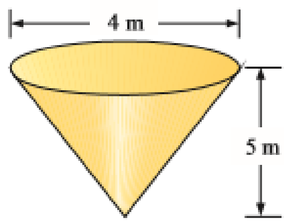
They should find a way to calculate its volume and find its capacity.

At the end of the activity, student should understand how to calculate volume and convert to capacity.

Exercise 10

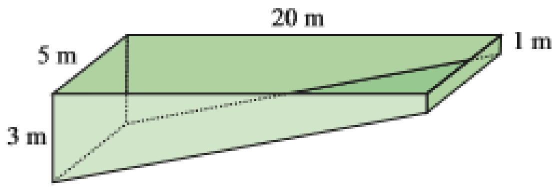
1. Convert
   a) 250mL to cm^3 b) $18.5\ m^3$ to kL c) $4100\ cm^3$ to L

2. A conical tank has a diameter] 4 m and height 5 m. How many litres of water could it contain?



A bowl is hemispherical in shape with an internal diameter of 30 cm. How many litres of water can it contain?

3. Find the number of litres of water that would be required to fill the swimming pool with the dimensions shown below.



4. If a sphere has a surface area of 100 cm^2, find to 1 decimal place:
   a) its radius b) its volume

Expected answers

1. a) 250 cm3 b) 18.5 kL c) 4.1 L

2. 20900L

3.7.07 L

4. 200,000 L

5. a) 8.92 cm b) 2970 cm^3

UNIT 3: GEOMETRY & TRIGONOMETRY

Math Secondary 1	**Unit 3: Geometry and trigonometry**
Learn About	**Key Inquiry Questions**
Learners should investigate angles of plane figures construct them manually as well as using computer technology to investigate enlargement which changes the size of an object but not the shape and understand the centre of enlargement and scale factor. They should understand and apply 3D coordinates of solids and their nets. Learners should investigate transformations including reflection, translation, reflection, congruency and rotation, and understand the mirror line as the angle of reflection and vectors as the distance and direction of translation. They should understand that after enlargement, the enlarged shape is congruent and similar to the object, and when there is a negative scale factor the image is on the opposite side to the centre of enlargement Learners should use a compass to investigate directions and understand bearings and how to calculate back bearings and should know about angles of depression and elevation and act them out in pairs. They should investigate the relationship between sine, cosine and tangent as ratios of side lengths of a right-angled triangle and apply formulae to solve problems. They should find out how to identify trigonometric ratios of special angles (zero, 30, 45, 60, 90) and their multiples without using tables.	• How do you construct a point, line and triangle? • How do you construct a scale drawing? • How can you use bearing (with or without a compass) to determine your direction and position in a desert/jungle?

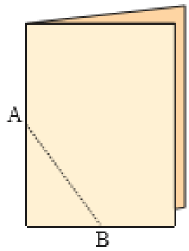
<table>
<tr><th colspan="3">Learning Outcomes</th></tr>
<tr><th>Knowledge and understanding</th><th>Skills</th><th>Attitudes</th></tr>
<tr>
<td>Understand how to use:
• 3D Coordinates
• Angles of plane figures
• Geometrical constructions
• Scale drawing and bearing
• Angles of depression and elevation.
• Reflection, congruency and rotation.
• Relation and mapping
• Translation as a transformation
• The relationship between sine, cosine,</td>
<td>• Use geometrical instruments to construct solids on a three - dimensional coordinate system.
• Use trigonometry to calculate heights of buildings, width of rivers, etc.
• Plan and carry out investigations using scale drawings, and trigonometry
• Use compass bearings to determine direction
• Identify trigonometric ratios of special angles (zero, 30, 45, 60, 90) and their multiples without using tables.</td>
<td>• Appreciation of 3D constructions and calculations using trigonometry.</td>
</tr>
<tr><td colspan="3">Contribution to the competencies
<u>Critical thinking:</u> in solving problems
<u>Co-operation and communication</u>: in group work</td></tr>
<tr><td colspan="3">Links to other subjects
Geography and mapping</td></tr>
</table>

The teacher should guide a discussion on the definitions on page 40 and 41 in the student's book. Divide students into groups of 4. Encourage discussion and creativity.

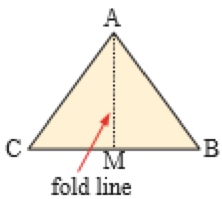
Check for understanding of angles, lines, points etc. and in the activities in Exercise 1.

Isosceles Triangles:
Task: Making an Isosceles Triangle

Working in pairs, get a clean sheet of paper.
Fold it down the middle. Draw a straight line AB as shown in the diagram below. Then, with the 2 sheets pressed tightly together, cut along line AB through both sheets.



Keep the triangular piece of paper. When you unfold it, you should obtain the isosceles triangle ABC shown.



Discussion

In the triangle ABC above, explain why:

- Angle ABC=Angle ABC
- M is the midpoint of BC
- AM is at right angles to BC

From the task above, we can conclude that in any iscosceles triangle,

- The base angles are equal
- The line joining the apex to the midpoint of the base is perpendicular to the base.

In exercise 2, students should be able to measure angles correctly and differentiate between types of triangles.

Investigation:

Students should work in pairs. Each pair should draw a triangle ABC. Mark the measurement of each angle. Students should discover that the angles in a triangle add up to 180^0.

Each pair should exhibit creativity and critical thinking skills

Exercise 3

Students should continue the discussion in pairs but write their answers individually in their exercise books. Teacher should facilitate discussions by walking around and prompting learners on reasons for their answers.

Expected answers

1.a) $x = 48$ b) $x = 42$ c) $x = 48$

2. a) $a = 55$ b) $a = 125$ c) $a = 146$ d) $a = 69$ e) $a = 55$

f) $a = 130$

3. a) $x = 40$ b) $t = 45$ c) $a = 55$ d) $a = 49$ e) $a = 40$

f) $k = 55$ g) $n = 60$ h) $b = 45$ i) $r = 30$

4. a) Obtuse angles are angles greater than 90^0. Two obtuse angles in a triangle would add up to more than 180^0, which is not possible.

b) An obtuse angle and 90^0 would add up to 180^0. A triangle has 3 angles and they should all add up to 180^0.

5. a) $x = 40$ b) $x = 35$ c) $x = 40$ d) $x = 36$ e) $x = 11$

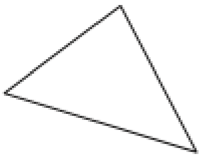
f) $x = 20$

Polygons

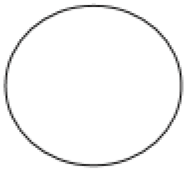
Discussion Questions

1. Which of these figures can be classified as a polygon?

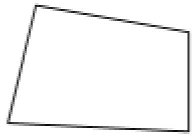
a)



b)



c)



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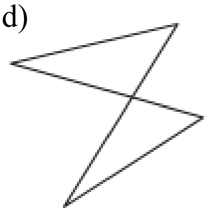
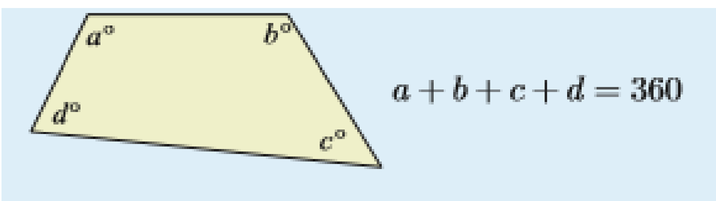


Investigation: Angles of a Quadrilateral

Students working in pairs should

- Draw any quadrilateral on the piece of paper. Label the vertices A, B, C and D on the inside of the quadrilateral. Cut out the quadrilateral.
- Tear off each of the 4 angles. Place them next to each other. The vertices should be meeting without overlapping as shown below. What do you notice?
- Draw other quadrilaterals and repeat the steps above.

Check that students are able to stick all the angles in order to get 360^0.
Students should also be able to draw triangles in the quadrilateral and use that to show the sum of the angles is 360^0



Transformation Geometry:

Translation

The teacher should guide the learners on a discussion of translation by demonstration. E.g. learners can move a number of steps to show translation.

Students should demonstrate understanding of

i. the translation vector and the fact that we always move in the *x* direction, then the *y* direction.

ii. that the shape does not change its shape or size after translation

Exercise 8:

Learners should discuss these questions in groups while still writing individual work and then compare answers amongst themselves.

1.a) $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ c) $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

2.a) $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ b) $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$ c) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$\

Rotation

The teacher should guide the learners on a discussion of rotation by demonstration. E.g. learners can move about a fixed point.

Students should demonstrate understanding of the **angle of rotation** and the **centre of rotation**

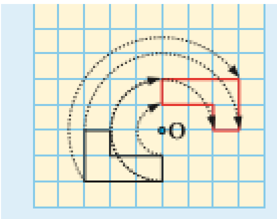
Task: Rotations using tracing paper.

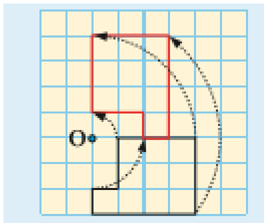
Students should work in pairs to do this activity. Look out for organizational, cooperation, analytical and critical thinking skills

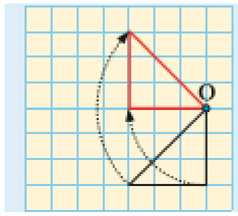
Expected answers:

$A^1(1,-1)$ $B^1(3,-1)$ and $C^1(3,-3)$

$A^2(-2,3)$ $B^2(-3,3)$ and $C^2(-3,5)$







Students should then attempt the questions in Exercise 9 still working in pairs and compare answers. Encourage discussion of the answers.

Reflection

The teacher should guide the learners on a discussion of reflection by demonstration.

Students should demonstrate understanding of the **mirror line** and that the object and image points are equidistant from the mirror line.

Task

Students should work in pairs to do this activity. Look out for organizational, cooperation, analytical and critical thinking skills

Expected solutions

1. Mirror line is the y −axis
   $$A'(2,-2)\ B'\ (6,-2)\ and\ C'\ (\ 6,-5)$$

6. P'$(2,-2)$ Q' $(2,2)$ R'$(-2,2)$ s' $(-2,-2)$
   P'$(-2,2)$ Q'$(-2,-2)$ R'$(2,-2)$ and S'$(2,2)$

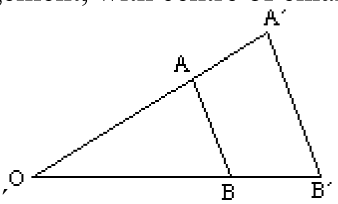
Students should then attempt the questions in Exercise 9 still working in pairs and compare answers. Encourage discussion of the answers.

Enlargement

An enlargement is a transformation where an object maps to an image of the same shape but different size. The object and the image are said to be similar. An enlargement requires a centre of enlargement and a scale factor.

Notation

E is an enlargement, with centre of enlargement O and scale factor μ.



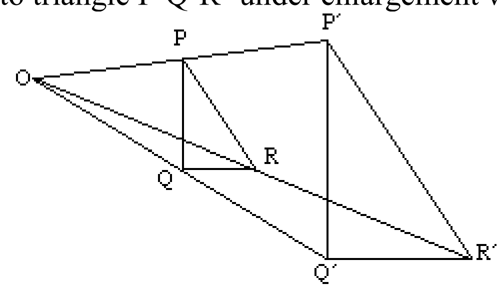
E: AB → A′B′

$$\mu = \frac{\text{Distance from centre to image point}}{\text{Distance from centre to corresponding object point}} = \frac{OA'}{OA} = \frac{OB'}{OB}$$

$$\mu = \frac{\text{A length on the image}}{\text{The corresponding length on the object}} = \frac{A'B'}{AB}$$

Properties of Enlargement

Triangle PQR maps to triangle P′Q′R′ under enlargement with centre O.



Lines and their images are always parallel.
e.g. PQ is parallel to P′Q′
Angle size is invariant.

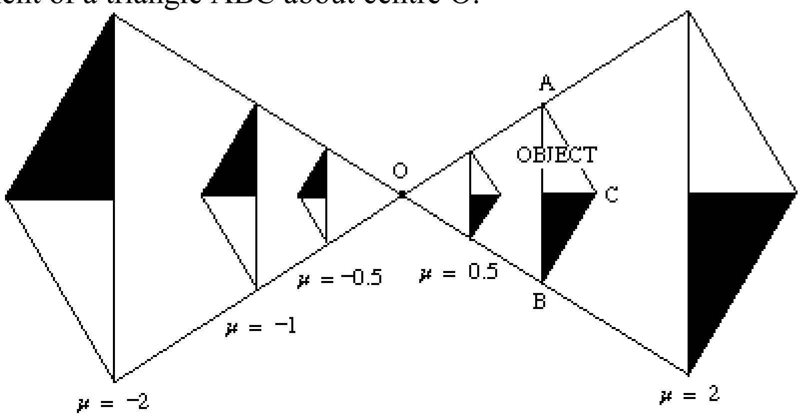
e.g. ∠ PQR = ∠ P′Q′R′
The centre of enlargement is the only invariant point.

Length and area are not invariant, except when $m = 1$ or -1.
i.e. Enlargement is not an isometry.

Enlargement is a direct transformation.
i.e. $\triangle PQR$ and $\triangle P'Q'R'$ are both anti-clockwise.
If μ is the scale factor for length, $m\mu^2$ is the scale factor for area.

Different Scale Factors

The diagram below shows the effect of a variety of scale factors on the enlargement of a triangle ABC about centre O.



- If the scale factor is positive, both the object and the image are on the same side of the centre.
- If the scale factor is negative, the object and the image are on opposite sides of the centre.
  The image is inverted.

Location of the Centre of Enlargement

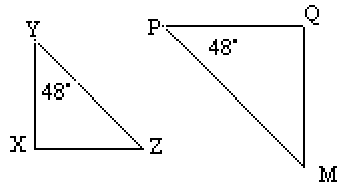
Given a figure and its image, to find the centre of enlargement:
1. Join up a point and its image.
2. Repeat for another point and its image.
3. The centre of enlargement is the intersection of these lines.

Similar Figures
Figures are similar if they have the same shape.
- Similar figures can be mapped onto one another by an enlargement or by a combination of reflection, rotation or translation and an enlargement.
- The corresponding angles of similar figures are equal.

- The corresponding sides are proportional to one another.

Type 1

△YXZ is similar to △PQM as they have corresponding angles equal.

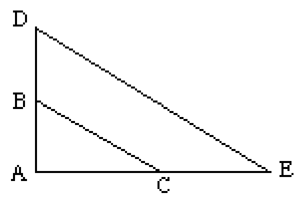
∠P = ∠Y ∠Q = ∠X ∠M = ∠Z	

△YXZ can be mapped to △PQM by a combination of transformations.

Scale factor = $\mu = \frac{PQ}{YX} = \frac{QM}{XZ} = \frac{PM}{YZ}$

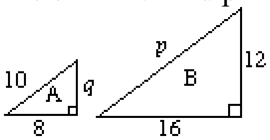
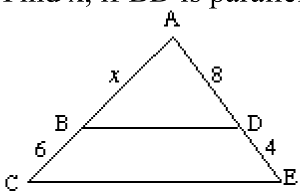
Type 2

△ABC is similar to △ADE because:

∠A is common, ∠B = ∠D ∠C = ∠E (DE is parallel to BC)	

△ABC can be mapped onto △ADE by an enlargement.

Scale factor = $\mu = \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

Examples	Answer
(a) (i) The two triangles A and B are similar. Find p and q.  	(a) (i) For A $\rightarrow$ B , scale factor = 2 p = 2 × 10 p = 20 2 x q = 12 q = 6
(ii) The area of triangle A is 24 units^2. What is the area of triangle B?	(ii) Scale factor for area = $\mu^2 = 4$ Area of triangle B = 4 x area of triangle A = 4 x 24 = 96 units^2
(b) Find x, if BD is parallel to  CE	(b) For $\triangle ABD \rightarrow \triangle ACE$ $\frac{AE}{AD} = \frac{AC}{AB}$ $\frac{12}{8} = \frac{x+6}{x}$ 12x = 8(x + 6) 12x = 8x + 48 4x = 48 x = 12

1. Scale factor: 2, centre (0,0)
   A'(1,1.5), B'(2,1), C'(1.5,1.5)
2. Scale factor -2 , centre (0,0)
   W'(-1,-2),X'(-1,-3), Y'(-3,-3) and Z'(-3,-2)

Students should then attempt the questions in Exercise 10 still working in pairs and compare answers. Encourage discussion of the answers.

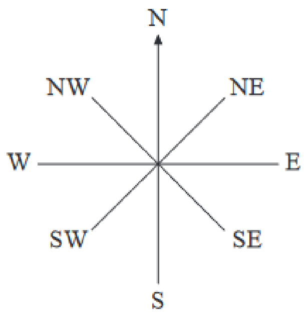
Bearings

Students should be in groups of 4/5. Generate a class discussion on bearing and its meaning. Demonstration can be used to explain that bearing is measured in a clockwise direction from the North line. Use students to explain bearing of one student to the other.

Students should understand the examples given. Encourage them to sketch diagrams and label the north line and then answer the questions.

Task

In pairs, copy and complete the table.



Compass points	Bearing
N	
NE	
E	
SE	
S	
SW	
W	
NW	

Still in groups, let students attempt exercise 11. Check for understanding. Students should be encouraged discuss amongst themselves, try various approaches and finally agree on a particular answer.

Exercise 12

Expected solutions

1 a) 180^0 b) 180^0 c) 45^0 d)135^0 e)45^0

2.

Direction	Bearing
N	000^0
NE	045^0
W	270^0
SW	225^0

7. Students should measure the angle for the tower to each of the point in a clockwise direction. For some, it will be easier to measure the angle in an anticlockwise direction and subtract form 360^0. Remember that answers should be in 3 digits.
8. 120^0, 300^0
9. 135^0

Scale drawings

Initiate class discussion on scale drawing and its importance. Use the examples to go over the key points (sketch and measure angle in a clockwise manner)

Exercise 13:

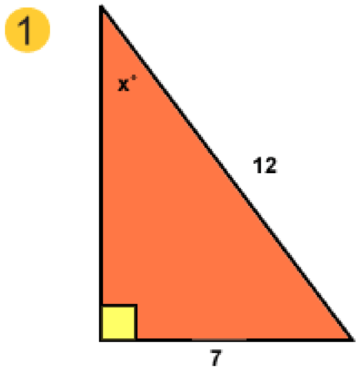
Students should work in groups but individually draw the scale diagrams. Guide the students to draw a sketch using the information given in the question.

Trigonometry

Exercise 14

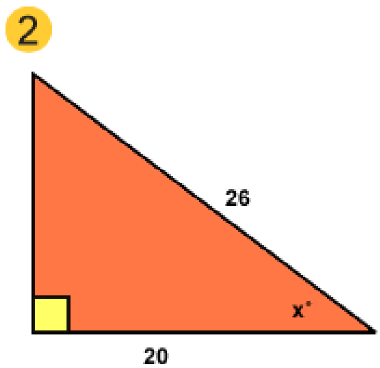
Use the same method in the following questions, but take care:
Sometimes you will use $\sin x^\circ$, sometimes $\cos x^\circ$ and sometimes $\tan x^\circ$.
Hint: Show all working, especially when you use a calculator.

Calculate x°. Give your answer to one decimal place.



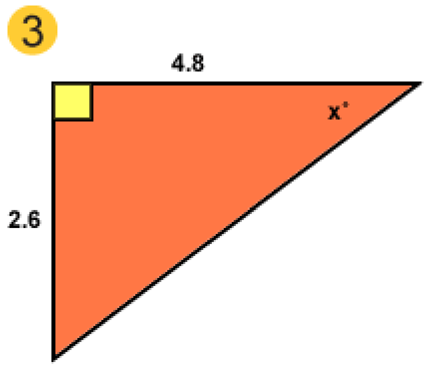
$$sin\ x^\circ = \frac{7}{12}$$
$$sin\ x^\circ = 0.5833...$$
$$x^\circ = 35.7^\circ$$

Calculate x°. Give your answer to one decimal place.



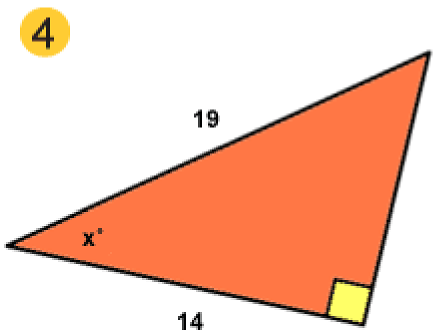
$$cos\ x^\circ = \frac{20}{26}$$
$$cos\ x^\circ = 0.769$$
$$x^\circ = 39.7^\circ$$

Calculate x°. Give your answer to one decimal place.



$$\tan x^\circ = \frac{2.6}{4.8}$$
$$\tan x^\circ = 0.54166$$
$$x^\circ = 28.4^\circ$$

Calculate x°. Give your answer to one decimal place.

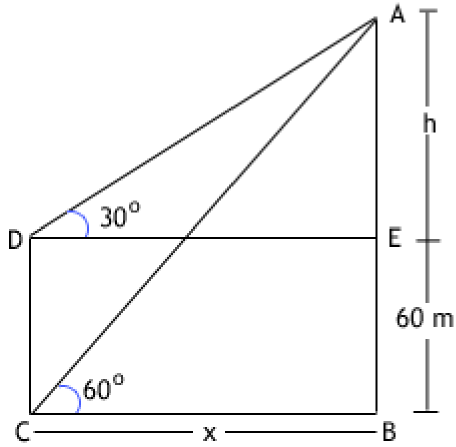


$$\cos x^\circ = \frac{14}{19}$$
$$\cos x^\circ = 0.7368$$
$$x^\circ = 42.5^\circ$$

Angles of Elevation and Depression

Exercise 15

1. The angles of elevation of the top of a tower from the top and bottom of a 60 m high building are 30o and 60o. What is the height of the tower?



Solution:

The height of the building DC = 60 m

Angle of elevation of the top of the tower from the top of the building = 30°

Angle of elevation of the top of the tower from the bottom of the building = 60°

Let, x be the distance between building and tower.

tan D = opposite side/adjacent side

[Choose an appropriate trigonometric ratio.]

From ΔADE, tan 30° = AE / DE

$$0.5773 = hx$$

$$x = h0.5773$$

[Multiply both sides with x / 0.5773.]

From ΔABC, $tan\ 60o = (h+60) / x$

$$1.732 = (h+60)x$$

[Substitute the values.]

$$x = (h+60)1.732$$

[Multiply both sides with x / 1.732.]

From steps (7) and (10), $h / 0.5773 = (h+60) / 1.732$

$$3h = h + 60$$

[Multiply both sides with 1.732.]

$$h = 602 = 30$$

[Divide both sides by 2 and simplify.]

The height of the tower = $h + 60$

$$= 30 + 60 = 90$$

[Substitute and simplify.]

The height of the tower is 90 m.

1. $x = 61\,m$
2. $x = 2256.9\,m$
3. $x = 15.9\,ft.$
4. $x = 189.9\,m$
5. Height of kite: 78 ft.; Ground distance from man to kite: 61.6 ft
6. $x = 59.3\,m$
7. The plane must climb at an angle greater than 16.7°
8. $\theta = {}^\circ 68.2$
9. Do not do this problem 10. Legs: 5.3 &
10. 7 ; Hypotenuse: 12
11. First ship: 2399.2 m; Second ship: 4343.4 m; Distance between ships: 1944.2 m
12. 36.5 cm

UNIT 4: ALGEBRA

Expressions, Equations and Inequalities

Math: Secondary 1	**Unit 4: Algebra**
Learn About	**Key Inquiry Questions**
Learners should would in pairs and groups to use simplification, single, double and squared brackets, substitution, factorization, expansion and the application of BIDMAS to solve a range of problems. They should apply simultaneous linear equations and formulae and solve inequalities in a range of situations. They should use numerical examples in simplification, of brackets, substitution and expansion, *e.g. 5 (6 – 2) = 30 –10 = 20.* Learners should work in pairs and groups to form their own meaningful equations and solve them as part of real world applications of mathematics including formulae and equations that apply to science such as density=mass/volume and speed=distance/time.	• What are variables/ unknowns stand for in an algebraic expression? • Why do we need brackets in expanding and factorising algebraic expressions? • How do we form simultaneous equations? • How do we derive inequality and solve it?

Learning Outcomes		
Knowledge and understanding	**Skills**	**Attitudes**
Understand: • Simplification, brackets, substitution • Factorization and expansion, • Simultaneous linear equations • Formation and solution of inequalities • Formulae and equations	• Use algebra to solving problems involving expansion, factorization of algebraic expressions as well as simultaneous equations. • Represent and formulate various mathematical relations using functions and mappings.	• Appreciate the importance of using algebra to find an unknown/variable

<table>
<tr><td>Functions</td><td>Design and carry out investigations using algebraic expression and formulae to solve multi step problems</td><td></td></tr>
<tr><td colspan="3">Contribution to the competencies
Critical thinking: in solving problems
Co-operation and communication: in group work</td></tr>
<tr><td colspan="3">Links to other subjects
The Sciences and ICT</td></tr>
</table>

Simplification of expressions

Teacher should encourage discussion on how letters can be used to write a statement in algebraic form.

Investigation: Work in groups.

Find out as much as you can about the early history of algebra. You might consider:

a) Ahmes Papyrus (Egyptian c. 1700 BC)
b) Diophantus (Greek c. AD 250)
c) Mohammed ibn Musa al-Khowarizmi (Arab c. AD 825)
d) Bhaskara (Hindu c. AD 1150)

Present your findings to the whole class

Exercise 1: In groups,

1. Write in words, the meaning of:
   a) $c - b$
   b) $2a + d$
   c) $3(l + m)$
   d) $p^2 + q$
   e) $a + b$
2. Write the following as an algebraic expression:
   a) The sum of p and the square of q
   b) three times the product of a and b
   c) twice *n* minus *m*
   d) The square of the sum of c and d
   e) 10 less than c
3. Write in words the meaning of:
   a) $A = 2x + 2y$
   b) $X = \frac{2c}{d}$
   c) $B = 3a^2 + d$
   d) $f = 4x - y$

Expected answers

1. a) the difference of c and b

b) the sum of twice the value of a and d

c) the sum of l and w multiplied by 3

d) the sum of the square of p and q

e) the sum of a and b

2.a) $(p+q)$

b) $3(a+b)$

c) $(m-2n)$

d) $(c+d)^2$

e) $(c-10)$

3. **The teacher should guide discussion on the various ways of writing the statements. Let students be as creative as possible as long as it makes mathematical sense.**

a A is the sum of twice x and twice y

b) Twice c divided by d is X (the quotient of twice c and d is X)

c) B is the sum of thrice the square of a and d

d)the difference of four times x and y is f (four times x less y is equal to f.)

Simplifying algebraic expressions

Exercise 2

Simplifying Algebraic expressions

Learners should be in groups. Guide discussion on how the statements should be written. Any letters can be used for question 1 & 2.

In groups, write the following statements as algebraic expressions and simplify where necessary.

Simplify each of the following by collecting like terms:

(a) $4a + b + 2a$

(b) $4b + 2c + 6b + 3c$

(c) $4a + 5b - a + 2b$

(d) $14p + 11q - 8p + 3q$

(e) $6x - 4y + 8x + 9y$

(f) $11x + 8y + 3z - 2y + 4z$

(g) $16x - 8y - 3x - 4y$

(h) $11y + 12z - 10y + 4z + 2y$

Expected answers

1. 25t+5n

2. 9g+7s

3. a) $6a + b$

b) $10b + 5c$

c) $3a + 7b$

d) $6p + 14q$

e) $14x + 5y$

f) $11x + 6y + 7z$

g) $13x - 12y$

h) $3y+16z$

Algebraic Substitution

Exercise 3:

Guide students through the example and give your own examples then in pairs, let them answer the questions. Working out should be shown clearly.

Review of working with negative numbers may be necessary.

Working in pairs, answer the following questions:

1. Given that $p = 2z - m$, find
   i) p when $z = 10$ and $m = 2$
   ii) p when $z = -5$ and $m = 3$
   iii) m when $p = 20$ and $z = 2$

2. If $p = 4, q = -2$ and $r = 3$, find the value of
   i) $3q - 2r$
   ii) $2pq - r$
   iii) $\dfrac{p - 2q + 2r}{p + r}$

3. If $a = 3, b = -2$ and $c = -1$, find the value of
   i) b^2
   ii) $ab - c^3$

4. If $p = 4, q = -3$ and $r = 2$, find the value of
   i) $\sqrt{p - q + r}$
   ii) $\sqrt{p^2 + q^2}$

5. A formula states:

   $$y = 4x - 5$$

a) Calculate y if $x = 3$.
b) Determine x if $y = 23$.
c) Determine x if $y = 8$.

Expected answers

1. i)18 ii) -13 iii) -16

2. i) -12 ii) -19 iii) 2

3. i) 4 ii) -5

4. i) 3 ii) 5

5. a) 7 b) 7 c) $3\frac{1}{4}$

Expansion of algebraic expressions

Teacher should guide the students through the example. Encourage the students to show their working clearly.

"Expanding" means **removing the** ( ) ... but we have to do it the right way!

( ) are called "parentheses" or "brackets"

Whatever is inside the ( ) needs to be treated as a "package".

So when multiplying: multiply by everything inside the "package".

Example: Expand 3 × (5+2)

We can complete the calculation:

$$3 \times (5+2) = \mathbf{3 \times 5 + 3 \times 2}$$
$$= 15 + 6$$
$$= 21$$

In Algebra putting two things next to each other usually means to multiply.

So **3(a+b)** means to multiply **3** by **(a+b)**

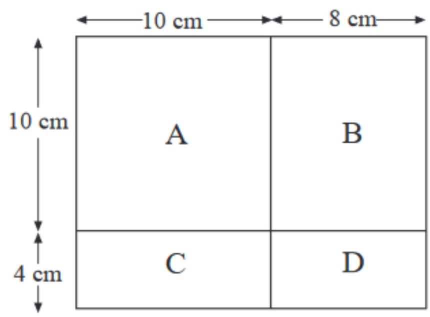
Here is an example of expanding, using variables **a**, **b** and **c** instead of numbers:

And here is another example involving some numbers. Notice the "·" between the 3 and 6 to mean multiply, so **3·6 = 18**:

Multiplying negatives has special rules: a negative times a positive gives a negative, but multiplying two negatives gives a positive:

Exercise 4: In pairs, work out the following questions.

1. a) The diagram shows a rectangle 18 cm long and 14 cm wide. It has been split into four smaller rectangles, A, B, C and D.
   i) Write down the area of each of the small rectangles. One has been done for you.

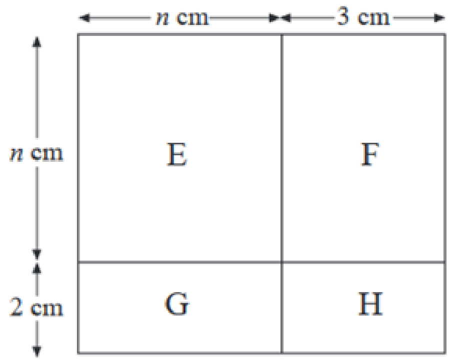


Area of Rectangle C = 40 cm^2.

   ii) What is the area of the *whole* rectangle?
   iii) What is 18 x 14?

a) The diagram shows a rectangle $(n+3)$ cm long and $(n+2)$ cm wide. It has been split into four smaller rectangles.

a) Write down a number or an expression for the area of each small rectangle. One has been done for you.



Area of Rectangle F = $3n$ cm^2.

iii) What is $(n+3)(n+2)$ multiplied out?

2. Expand:

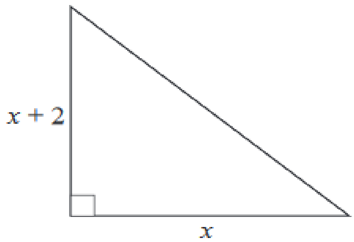
a. $(x + 6)$	f. $8\,(3x - 9)$
b. $3\,(x - 4)$	g. $(-2)(x - 4)$
c. $5\,(2x + 6)$	h. $(-3)(8 - 2x)$
d. $7\,(3x - 4)$	i. $5\,(3x - 4)$
e. $3\,(2x + 4)$	j. $9\,(2x + 8)$

3. Deng writes $3\,(4x - 8) = 12x - 8$. Explain why his expansion is *not* correct.

4. Write down an expression for the area of this triangle, that:

a) contains brackets,

b) does *not* contain brackets.



5. Expand each of the following expressions by multiplying out the brackets.

   a) $(x+4)(x+3)$ c) $(x+2)(x+4)$ e) $(x+1)(x+5)$

   b) $(x+6)(x-1)$ d) $(x-4)(x+2)$ f) $(x-3)(x+2)$

6. Simplify each of the following expressions.
   a) $(x+2)(x+4)+(x+1)(x+2)$
   b) $(x+3)(x+7)+(x-1)(x+5)$
   c) $(x+6)(x+2)-(x-2)(x+3)$
   d) $(x-4)(x-8)-(x-1)(x-9)$

7. Expand.
   a) $(x+1)^2$
   b) $(x-2)^2$
   c) $(x+3)^2$
   d) $(x+5)^2$
   e) $(x-7)^2$
   f) $(x-8)^2$
   g) $(x+\ 10)^2$
   h) $(x-12)^2$
   i) $(x+4)^2$
   j) $(2x+3)^2$
   k) $(4x-7)$
   l) $(3x+2)^2$
   m) $(4x+1)^2$
   n) $(5x-2)^2$
   o) $(6x-4)^2$

8. Expand:
   b) $(x+1)^3$ b) $(2x+1)^3$ c) $(x-5)^3$

Expected answers

1. **a)** $4x+24$ **b)** $3x+2$ **c)** $10x+30$

d) $21x - 28$ **e)** $6x + 12$ **f)** $24x - 72$

g) $-2x + 8$ **h)** $-24 + 6x$ **i)** $15x - 20$

j) $18x + 72$

2. Allow students to explain why Jordan is wrong. The correct answer is $12x - 24$

3. a) $\frac{1}{2}x(x+2)$ **b)** $\frac{1}{2}x^2 + x$

4. **a)** $x^2 + 7x + 12$ **b)** $x^2 + 6x + 8$ **c)** $x^2 + 6x + 5$

d) $x^2 + 5x - 6$ **e)** $x^2 - 2x - 8$ **f)** $x^2 - x - 6$

5. a) $2x^2 + 9x + 10$ **b)** $2x^2 + 14x + 16$

c) $7x + 18$ **d)** $-2x + 23$

6. **a)** $x^2 + 2x + 1$ **b)** $x^2 - 4x + 4$ **c)** $x^2 + 6x + 9$
d) $x^2 + 10x + 25$ **e)** $x^2 - 14x + 49$ **f)** $x^2 - 16x + 64$
g) $x^2 + 20x + 100$ **h)** $x^2 - 24x + 144$ **i)** $x^2 + 8x + 16$

j) $4x^2 + 12x + 9$ **k)** $16x^2 - 56x + 49$ **l)** $9x^2 + 12x + 4$

m) $16x^2 + 8x + 1$ **n)** $25x^2 - 20x + 4$ **o)** $36x^2 - 48x + 16$

7. This is an extension question for the learners who understand expansion.

Learners should know that $(x+1)^3 = (x+1)(x+1)^2$ **come up with ways to expand this expressions**

a) $x^3 + 3x^2 + 3x + 1$ **b)** $8x^3 + 12x + 6x + 1$

8. a) Rectangle A: 100cm^2 Rectangle B: 80 cm^2 Rectangle D: 32cm^2

ii) 100+80+40+32=252 cm^2

iii) 18x14=252

b) i) **Rectangle E**: n^2 **cm²**

Rectangle G: $2n$ **cm²**

Rectangle H: 6cm²

ii) $n^2 + 5n + 6$

Learners should see the relationship between the expansion of $(n+3)(n+2)$and the total area of the rectangle, found by adding $n^2 + 2n + 3n + 6$

Factorizing algebraic expressions

Factorizing is the process of simplifying an algebraic expression by removing the **'highest common factor'** and placing it outside a set of parentheses which contains the remainder of the expression.

For example:

$x^2 + x = x(x + 1)$

In the example above, the highest common factor is *x*. The highest common factor is the highest number (including numerals and letters) which can be found in *every* term of the algebraic expression.

Teacher should encourage students to discuss as they individually write solutions in their exercise books. Circulate around the class to facilitate discussions in the various groups.

Exercise 5

1. Factorise:

a) $4x - 2$
b) $6x - 12$
c) $5x - 20$
d) $4x + 32$
e) $6x - 8$
f) $8 - 12x$
g) $21x - 14$
h) $15x + 20$
i) $30 - 10x$

2. Factorise:
    a) $x^2 + 4x$
    b) $x^2 - 3x$
    c) $4x - x^2$
    d) $6x^2 + 8x$
    e) $9x^2 + 15x$
    f) $7x^2 - 21x$
    g) $28x - 35x^2$
    h) $6x^2 - 14$
    i) $5x^2 - 3x$

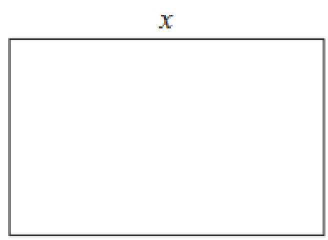
3. Factorise:
    i) $x^3 + x^2$
    ii) $2x^2 - x^3$
    iii) $4x^3 - 2x^2$
    iv) $8x^3 + 4x^2$
    v) $16x^2 - 36^3$
    vi) $4x^3 + 22x^2$
    vii) $16x^2 - 6x^3$
    viii) $14x^3 + 21x^2$
    ix) $28x^3 - 49x^2$

4. a) Expand $(x + 5)(x - 5)$
    a) Factorise $x^2 - 25$
    b) Factorise each of the following.
        i. $x^2 - 49$
        ii. $x^2 - 64$
        iii. $x^2 - 100$
        ii. $x^2 - a^2$
        iii. $x^2 - 4b^2$

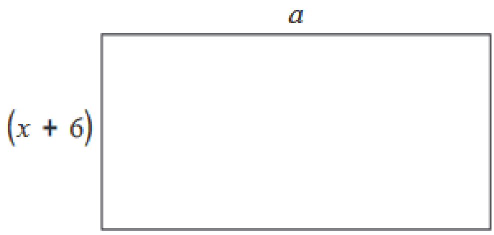
5. Factorise:
    a) $x^2 + 7x + 12$
    b) $x^2 + 8x + 7$
    c) $x^2 + 11x + 18$
    d) $x^2 + 12x + 27$
    e) $x^2 + 17x + 70$
    f) $x^2 + 6x + 8$

Extension questions

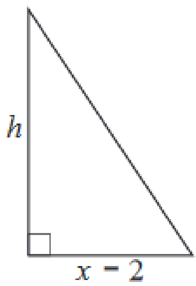
6. The area of the rectangle shown is $x^2 - 5x$. Express *a* in terms of *x*.



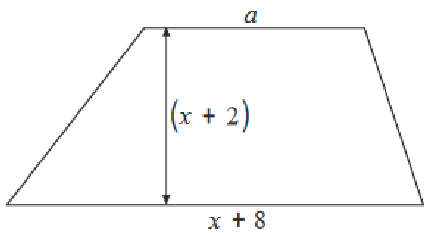
7. The area of the rectangle shown is $x^2 + 11x + 30$. Express a in terms of x.



8. The area of the triangle shown is $\frac{1}{2}x^2 + \frac{3}{2}x - 5$. Express h in terms of x.



9. The area of the trapezium shown is $\frac{1}{2}x^2 + 10x + 18$. Determine a.



Expected answers

1. a) $2(x - 2)$ b) $6(x - 2)$ c) $5(x - 4)$

d) $4(x + 8)$ e) $2(3x - 4)$ f) $4(2 - 3x)$

g) $7(3x - 2)$ h) $5(3x - 4)$ i) $10(3 - x)$

2. a) $x(x+4)$ b) $x(x-3)$ c) $x(4-x)$

d) $2x(3x+4)$ **e)** $3x(3x+5)$ **f)** $7x(x-3)$

g) $7x(4-5x)$ **h)** $3x(2x-7)$ **i)** $5x(5x-3)$

3. a) $x^2(x+1)$ **b)** $x^2(2-x)$ **c)** $2x^2(2x-1)$

d) $4x^2(2x+1)$ **e)** $4x^2(4-9x)$ **f')** $2x^2(2x+11)$

g) $2x^2(8-3x)$ **h)** $7x^2(2x+3)$ **i)** $7x^2(4x-7)$

4. **a)** x^2-25

b) $(x+5)(x-5)$

c) **i)** $(x+7)(x-7)$

ii) $(x+8)(x-8)$

iii) $(x+10)(x-10)$

iv) $(x+a)(x-a)$

v) $(x+2b)(x-2b)$

5. Use the examples given to have a discussion with the learners on how to factorise these questions. Learners should be made aware that the order of the product does not matter. E.g. $(x+5)(x-5)$ **is the same as** $(x-5)(x+5)$

a) $(x+4)(x+3)$

b) $(x+7)(x+1)$

c) $(x+2)(x+9)$

d) $(x+9)(x+3)$

e) $(x+10)(x+7)$

f) $(x+4)(x+2)$

6. $ax = x^2 - 5$

7. $x^2 + 11x + 30 = a(x+6)$

8.
$$A = \frac{1}{2}x^2 + \frac{3}{2}x - 5$$
$$\frac{1}{2}(x-2)h = \frac{1}{2}x^2 + \frac{3}{2}x - 5$$

$a = (x-5)$

$(x+6)(x+5) = a(x+6)$
$a = (x+5)$

$(x-2)h = x^2 + 3x - 10$
$h(x-2) = (x+5)(x-2)$

$h = (x+5)$

9. $A = \frac{1}{2}(a + x + 8)(x+2) = \frac{1}{2}x^2 + 10x + 18$
$a = 10$

Equations

Basic Terminology

Math with letters is really just an extension of math without letters. Algebra simply makes it easier to talk about something with an unknown value. Mathematicians have agreed to call the letter that is used to represent an unknown quantity a variable. It's still called a variable even when it represents a single specific number.

The '3' in $3x + 2 = 17$ is called the coefficient, while the '2' and '17' are called constants and we can refer to them as constant terms. Any terms that are multiplied by the same variable or combination of variables are like terms. $3y$ and $10y$ are like terms, as are $3xy$ and $23xy$. Compare those to $3x$ and $7y$, which are not like terms and can't be combined.

Definition of Algebraic Equation

There are a few rules we must observe:

- an algebraic equation must contain a variable
- the variable must be multiplied by a coefficient that is not zero
- there should be an equal sign

Is our equation, $3x + 2 = 17$, an algebraic equation?

Yes! It has a variable multiplied by a non-zero coefficient (3) and has an equal sign, so it meets our requirements.

Solving Single Variable Equations

'Solving' an algebraic equation just means manipulating the equation so that the variable is by itself on one side of the equation and everything else is on the other side of the equation. Once 'everything else' is simplified, the equation is solved.

The most simple algebraic equation you could have would be something like $x=5$, which is both an algebraic equation and its own solution.

Let's try something a little harder: $y + 5 = 10$

How can we get the y by itself? Why, get rid of the 5 of course! Only it's not quite that easy. Whatever we do to one side of the equation we need to do to the other as well. What do we need to do the left side to get rid of that pesky 5?

Subtract 5 from both sides of the equation. Doing so makes our equation become:

$y + 5 - 5 = 10 - 5$

This is kind of clunky, so let's combine like terms.

$y + (5 - 5) = (10 - 5)$

5 - 5 = 0 and 10 - 5 = 5, so our equation becomes:

$y = 5$

Exercise 6

1. Solve the following equations.
2. Make up some equations of your own for your friends to try.

Simultaneous Equations

Exercise 7

1. Solve the following equations.

a) $2x - y = 1$
$2x + 2y = 10$

b) $2x + 3y = 8$
$5x - 2y = 1$

c) $3x + 3y = 7$
$4x + 3y = 2$

d) $5x - y = 15$
$x + y = -3$

e) $3x - y = 6$
$2x + 5y = -13$

f) $3x + 5y = 1$
$x - y = -5$

g) $2x - 5y = 3$
$3x + 2y = 14$

h) $2x + 3y = 6$
$3x + 2y = -1$

i) $3x - y = 18$
$x + 2y = -1$

2. Make up some equations of your own for your friends to try.

Expected answers

1	a) x=3	b) x=9	c) x=5	d) x= 14	e) x=1
	f) x=2				
2	a) $x = -2$	b) $x = -2$	c) $x = -4$	d) $x = -4$	e) $x = -4$
	f) $x = -2$	g) $x = 5$	h) $x = -2$	i) $x = 6$	
3	a) $x = 2$	b) $x = 4$	c) $x = 1$	d) $x = -3$	e) $x = 3$
	f) $x = \frac{-20}{3}$				

UNIT 5: STATISTICS

<table>
<tr><th colspan="3">Math: Secondary 1</th><th>Unit 5: Statistics</th></tr>
<tr><td colspan="3">Learn About</td><td>Key Inquiry Questions</td></tr>
<tr><td colspan="3">Learners should work in pairs and groups to collect, organise, tabulate and present raw data from a range situations and find average, mode and median. They should investigate the use of such data in sources such as newspapers, business and government publications, and evaluate the effectiveness and validity of the statistics presented.

They should construct frequency tables from given data, and interpret tables, commenting on their accuracy and effectiveness.

Learners should work in groups in a range of simple practical situations to investigate probability, find out how to calculate chance, and use this to predict one or more outcomes.

They should investigate theoretical and experimental probability and the occurrence of more than two or more events using a tree diagram.</td><td>- How do you organise and present data?
- How do you calculate average, mode and median?
- What is a frequency table and how do you construct it?
- What is probability?
- How useful is probability in predicting the occurrence of events?
- What is a tree diagram and how is it used in probability?</td></tr>
<tr><td colspan="4">Learning Outcomes</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td colspan="2">Attitudes</td></tr>
<tr><td>- Understand grouped data, average, mode and median
- Understand the compilation and use of frequency tables
- Understand the probability of events</td><td>- Collect statistical data and construct frequency tables
- Interpret and interrogate data
- Calculate probability in simple situations
- Use a tree diagram to investigate probability</td><td colspan="2">- Appreciate the importance of accurate data in statistics</td></tr>
<tr><td colspan="4">Contribution to the competencies</td></tr>
</table>

Critical thinking: in solving problems Co-operation and communication: in group work
Links to other subjects Physics, Chemistry, Biology, History, Geography

Statistics

What is statistics?

Statistics is a very broad subject, with applications in a vast number of different fields. In general one can say that statistics is the methodology for collecting, analyzing, interpreting and drawing conclusions from information. Putting it in other words, statistics is the methodology which scientists and mathematicians have developed for interpreting and drawing conclusions from collected data. Everything that deals even remotely with the collection, processing, interpretation and presentation of data belongs to the domain of statistics, and so does the detailed planning of that precedes all these activities.

From above, it should be clear that statistics is much more than just the tabulation of numbers and the graphical presentation of these tabulated numbers. Statistics is the science of gaining information from numerical and categorical data. Statistical methods can be used to find answers to the questions like:

- What kind and how much data need to be collected?
- How should we organize and summarize the data?
- How can we analyse the data and draw conclusions from it?
- How can we assess the strength of the conclusions and evaluate their uncertainty?

That is, statistics provides methods for

1. Design: Planning and carrying out research studies.
2. Description: Summarizing and exploring data.
3. Inference: Making predictions and generalizing about phenomena represented by the data.

Furthermore, statistics is the science of dealing with uncertain phenomenon and events. Statistics in practice is applied successfully to study the effectiveness of medical treatments, the reaction of consumers to television advertising, the attitudes of young people toward sex and marriage, and much more. It's safe to say that nowadays statistics is used in every field of science.

Example (Statistics in practice). Consider the following problems:

- Agricultural problem: Is new grain seed or fertilizer more productive?

- medical problem: What is the right amount of dosage of drug to treatment?
- political science: How accurate are the gallups and opinion polls?
- economics: What will be the unemployment rate next year?
- technical problem: How to improve quality of product?

Variables

A characteristic that varies from one person or thing to another is called a variable, i.e., a variable is any characteristic that varies from one individual member of the population to another. Examples of variables for humans are height, weight, number of siblings, sex, marital status, and eye color. The first three of these variables yield numerical information (yield numerical measurements) and are examples of quantitative (or numerical) variables, last three yield non-numerical information (yield non-numerical measurements) and are examples of qualitative (or categorical) variables.

Quantitative variables can be classified as either discrete or continuous.

Discrete variables. Some variables, such as the numbers of children in family, the numbers of car accident on the certain road on different days, or the numbers of students taking basics of statistics course are the results of counting and thus these are discrete variables. Typically, a discrete variable is a variable whose possible values are some or all of the ordinary counting numbers like 0, 1, 2, 3 . . . As a definition, we can say that a variable is discrete if it has only a countable number of distinct possible values. That is, a variable is discrete if it can assume only a finite numbers of values or as many values as there are integers.

Continuous variables. Quantities such as length, weight, or temperature can in principle be measured arbitrarily accurately. There is no invisible unit. Weight may be measured to the nearest gram, but it could be measured more accurately, say to the tenth of a gram. Such a variable, called continuous, is intrinsically different from a discrete variable.

Measures of central tendency

Descriptive measures that indicate where the center or the most typical value of the variable lies in collected set of measurements are called measures of center. Measures of center are often referred to as averages. The median and the mean apply only to quantitative data, whereas the mode can be used with either quantitative or qualitative data.

The Mode

The sample mode of a qualitative or a discrete quantitative variable is that value of the variable which occurs with the greatest frequency in a data set.

To obtain the mode(s) of a variable, we first construct a frequency distribution for the data using classes based on single value. The mode(s) can then be determined easily from the frequency distribution.

The Median

The sample median of a quantitative variable is that value of the variable in a data set that divides the set of observed values in half, so that the observed values in one half are less than or equal to the median value and the observed values in the other half are greater or equal to the median value. To obtain the median of the variable, we arrange observed values in a data set in increasing order and then determine the middle value in the ordered list.

The Mean

The most commonly used measure of center for quantitative variable is the (arithmetic) sample mean. When people speak of taking an average, it is mean that they are most often referring to.

Which measure to choose?

The mode should be used when calculating measure of center for the qualitative variable. When the variable is quantitative with symmetric distribution, then the mean is proper measure of center. In a case of quantitative variable with skewed distribution, the median is good choice for the measure of center. This is related to the fact that the mean can be highly influenced by an observation that falls far from the rest of the data, called an outlier. It should be noted that the sample mode, the sample median and the sample mean of the variable in question have corresponding population measures of center, i.e., we can assume that the variable in question have also the population mode, the population median and the population mean, which are all unknown. Then the sample mode, the sample median and the sample mean can be used to estimate the values of these corresponding unknown population values.

Probability

Probability is a measure of the weight of evidence, and is arrived at through reasoning and inference. In simple terms it is a measure or estimation of likelihood of the occurrence of an event. The word probability comes from the Latin word probabilitas which is a measure of the authority of a witness in a legal case. Some of the earlier mathematical studies of probability were motivated by the desire to be more profitable when gambling. Today however the practical uses of probability theory go far beyond gambling and are used in many aspects of modern life.

Activity

Students often hear and occasionally use statements of probability in their daily lives. They note the weather forecasts when they wonder whether a game will be held or school canceled. They also use more ambiguous general phrases such as not likely, no way, and probably but, as with much of everyday speech, there are many misuses of probability terminology and concepts.

Purpose: To introduce and develop the concept of probability.

Materials: Sack; marbles of two different colors - 100 of one color (blue), 25 of another color (green).

Procedure:

A. Announce to the class: Let's do an experiment. We will try to find out - without looking in the sack and counting - whether there are more blue or more green marbles in the sack.
B. Have four students draw five marbles each from the sack. (Make sure that the marbles are put back into the sack after each draw.)
C. Have every student record the numbers and colors of marbles for each of the four draws.

Questions to Ask and Answer:

1. On the basis of the first four draws how many marbles of each color are there in the sack? D. Let each student in the rest of the class draw five marbles each from the sack. (Be sure to put the marbles back in the sack after each drawing.)
2. What are the totals for each color of marble?
3. Do you think there were more marbles of one color than the other? Why?
4. If so, what do you think the ratio of one color to the other might be? G. Open the sack and count the number of marbles of each color.
5. What is the ratio of one color to the other color?

Conclusion

The probability of drawing a blue marble is four times greater than drawing a green one.

Discussion questions:

Use the questions to test students understanding of the probability number line.

Experimental probability

Activity: Complete in groups.

In groups let students perform the experiments.

Guide students to understand that the total probabilities of events is always 1. So that if 44 heads are obtained from the experiment, then 56 tails will have been obtained.

Exercise 2

1. When John makes an error in his work he crushes the sheet of paper into a ball and throws it at the waste basket. At the end of the day he has scored 16 hits into the basket and made 5 misses. Find the experimental probability that he scores a hit into the basket.

2. In the first round of competition Sasha recorded 77 hits out of 80 shots at her target.
   a) Use this result to estimate her chances of hitting the target
   b) On the next day in the final round of the competition, she scored105 hits out of 120 shots. Obtain the 'best 'estimate of her hitting the target with any shot.

   (best estimate is found by adding the numbers of hits and dividing the result by the total number of shots)

3. Paulo catches a 7.45 am bus to school. During a period of 79 days he arrives at school on time on 53 occasions. Estimate the probability that Paulo:
   a) arrives on time.
   b) arrives late.

4. Don threw a tin can into the air 180 times. From these trials it landed on its side 137 times. Later that afternoon he threw the same tin can into the air 150 times. It landed on its side 92 times.
   a) Find the experimental probability of 'landing on the side' for both sets of trials.
   b) List possible reasons for the differences in the results.

5. In a large county hospital, 72 girls and 61 boys were born in a month.
   a) How many children were born in total?

b) Estimate the probability that the next baby born at this hospital will be:

i) a girl ii) a boy

Expected answers

1. $\frac{16}{21} \approx 0.762$
2. ≈ 0.963 **b** $\frac{182}{200} = 0.91$

3.a) $\frac{53}{79} = 0.671$ b) $\frac{26}{79} = 0.329$

4. **a** ≈ 0.761, ≈ 0.613

   b Assuming the can does not change shape, the difference is almost certainly due to chance alone.

5. **a** 133 **b** **i** ≈ 0.541 **ii** ≈ 0.459

Sample space

Experiment 1: What is the probability of each outcome when a coin is tossed?

Outcomes: The outcomes of this experiment are head and tail.

Probabilities:

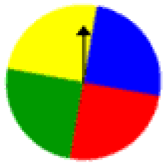
P(head) $= \frac{1}{2}$

P(tail) $= \frac{1}{2}$

Definition: The **sample space** of an experiment is the set of all possible outcomes of that experiment.

The sample space of Experiment 1 is: {head, tail}

***Experiment 2**:* A spinner has 4 equal sectors colored yellow, blue, green and red. What is the probability of landing on each color after spinning this spinner?



Sample Space: {yellow, blue, green, red}

Probabilities:

P(yellow) $= \frac{1}{4}$

P(blue) $= \frac{1}{4}$

P(green) $= \frac{1}{4}$

P(red) $= \frac{1}{4}$

***Experiment 3**:* What is the probability of each outcome when a single 6-sided die is rolled?

Sample Space: {1, 2, 3, 4, 5, 6}

Probabilities:

P(1) = 1

6

$$P(2) = \frac{1}{6}$$

$$P(3) = \frac{1}{6}$$

$$P(4) = \frac{1}{6}$$

$$P(5) = \frac{1}{6}$$

$$P(6) = \frac{1}{6}$$

Experiment 4*:* A glass jar contains 1 red, 3 green, 2 blue and 4 yellow marbles. If a single marble is chosen at random from the jar, what is the probability of each outcome?

Sample Space: {red, green, blue, yellow}

Probabilities:

$$P(\text{red}) = \frac{1}{10}$$

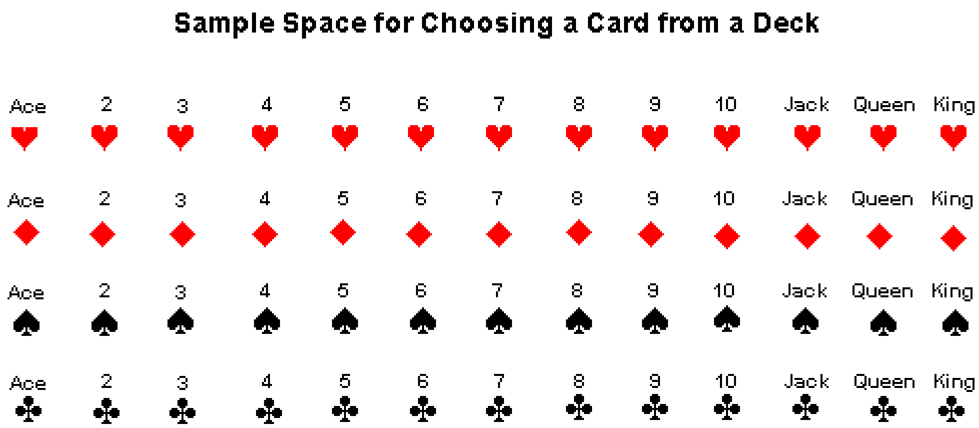
$$P(\text{green}) = \frac{3}{10}$$

$$P(\text{blue}) = \frac{2}{10} = \frac{1}{5}$$

$$P(\text{yellow}) = \frac{4}{10} = \frac{2}{5}$$

Summary: The sample space of an experiment is the set of all possible outcomes for that experiment. You may have noticed that for each of the experiments above, the sum of the probabilities of each outcome is 1. This is no coincidence. **The sum of the probabilities of the distinct outcomes within a sample space is 1.**

The sample space for choosing a single card at random from a deck of 52 playing cards is shown below. There are 52 possible outcomes in this sample space.



The probability of each outcome of this experiment is:

$$P(\text{card}) = \frac{1}{52}$$

The sum of the probabilities of the distinct outcomes within this sample space is:

$$\frac{52}{52} = 1$$

Discussion questions: Students should work in groups.

1.a) {1,2,3,4,5,6}

b) {hearts, diamonds, spades, clovers}

c) {Monday, Tuesday, Wednesday, Thursday}

d) {A, B, C, D}

2. a) 12

b) i) {2,4,6,8,10,12} ii) 6

c) i) {1,2,3,4} ii) 4

d)i) {4,8,12} ii) 3

Theoretical probability

Exercise 3

Students should be groups and should discuss the questions. The teacher should test understanding of experimental probability.

1. A marble is selected randomly selected from a box containing 5 green, 3 red and 7 blue marbles. Determine the probability that the marble is:
    a) red
    b) green
    c) blue
    d) not red
    e) neither green nor blue
    f) green or red

2. A carton contains eight brown and four white eggs. Find the probability that an egg selected at random is:
    a) brown
    b) white

3. In a class of 32 students, eight have one first name, nineteen have two first names, and five have three first names. A student is selected at random. Determine the probability that the student has:
    a) no first name

b) one first name
c) two first names
d) three first names.

4. An ordinary six sided doe is rolled once. Determine the chance of getting:
   a) a 5
   b) an odd number
   c) a number greater than 1
   d) a multiple of 2
5. In a club newsletter, 8 pages contain reports, 3 pages contain articles and 5 pages contain advertising. The newsletter is opened to a page at random. Determine the probability that it is:
   a) a report
   b) advertising
   c) not advertising
   d) a report or articles

Theoretical probability: Teacher should lead discussion on complementary events, sample spaces and finding probabilities.

Expected answers

1.

a) $\frac{1}{5}$ d) $\frac{4}{5}$

b) $\frac{1}{3}$ e) $\frac{1}{5}$

c) $\frac{7}{15}$ f) $\frac{8}{15}$

2. a) $\frac{2}{3}$ b) $\frac{1}{3}$

3. a) 0 b) $\frac{8}{32}$ c) $\frac{19}{32}$ d) $\frac{3}{32}$

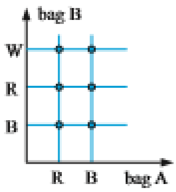
4. a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{5}{6}$ d) $\frac{1}{2}$

5. a) $\frac{1}{2}$ b) $\frac{5}{16}$ c) $\frac{11}{16}$ d) $\frac{11}{16}$

Using grids to find probabilities

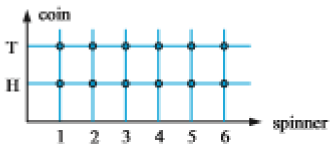
Exercise 4

1.



a $\frac{1}{6}$ **b** $\frac{1}{3}$
c $\frac{1}{3}$ **d** $\frac{2}{3}$

2. **a**



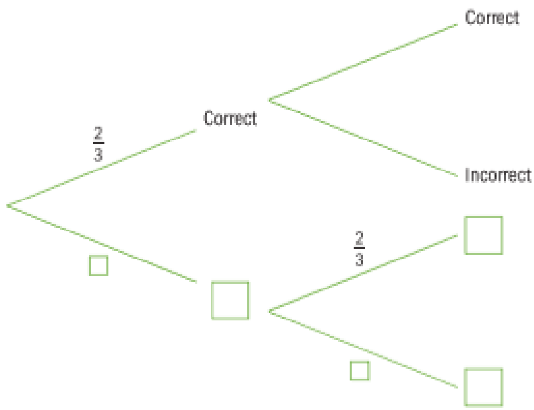
b 12

c **i** $\frac{1}{12}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{6}$ **v** $\frac{5}{6}$ **vi** $\frac{5}{12}$ **vii** $\frac{7}{12}$

Tree diagrams

Exercise 6: To be discussed in pairs.

1. Lizzie is attempting two exam questions. The probability that she gets any exam question correct is $\frac{2}{3}$.
   a) Copy and complete the diagram.
   b) What is the probability that she will get only one of them correct?
   c) What is the probability she will get at least one correct?



2. When John and Mark play in the hockey team the probability that John scores is $\frac{1}{3}$ and that Mark scores is $\frac{1}{2}$.
   Draw a tree diagram to illustrate this information and use it to find the probability that neither will score in the next game.

3. The probability of a day being windy is 0.6. If it is windy, the probability of rain is 0.4. If it is not windy, the probability of rain is 0.2.
   a) Draw a tree diagram to illustrate this information
   b) What is the probability of a given day being rainy?
   c) What is the probability of two successive days **not** being rainy?

4. There are equal numbers of boys and girls in a school and it known that $\frac{1}{10}$ of the boys and $\frac{1}{10}$ of the girls walk to school every day. Also $\frac{1}{3}$ of the boys and $\frac{1}{2}$ of the girls get a lift to school. The rest are boarders.
   Determine, using a tree diagram,
   a) the fraction of the school proportion that are girls who are boarders.
   b) the fraction of the school population that are boarders.

5. Garang does his shopping at the same time every Friday. He estimates that the probability that he has to queue at the supermarket checkout is $\frac{5}{6}$, and the probability that he has to queue at the post office is $\frac{1}{4}$. When he

does his shopping next Friday, what is the probability that he will queue at:

a) both places?

b) neither places?

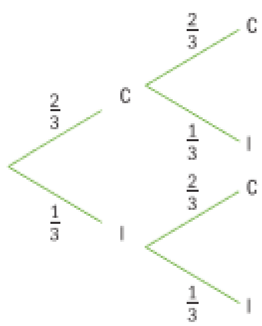
6. Nyoka has an 80% chance and Roger has a 50% chance of passing in their History examination. Find the probability that:
    a) both will pass
    b) Roger will pass and Helena will not
    c) neither will pass

7. In the month of November, the probability that it will rain tommorow is $\frac{2}{3}$, and the probability that I will carry my umbrella is $\frac{1}{4}$. What is the probabality that tomorrow:
    a) It will rain and that I will carry my umbrella
    b) It will rain and I will forget my umbrella

It will not rain and I will forget my umbrella.

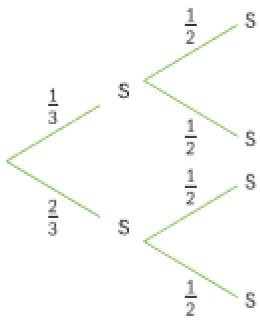
Expected answers

1. a)

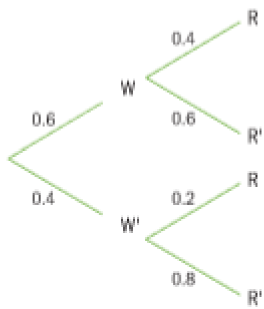


b) $\frac{4}{9}$ c) $\frac{8}{9}$

2.a) John Mark b) $\frac{1}{3}$



3. a) b)0.32 c)0.4624



4 a) $\frac{1}{5}$ b) $\frac{49}{120}$