South Sudan

Secondary Mathematics 1

Secondary Mathematics has been written and developed by Ministry of General Education and Instruction, Government of South Sudan in conjunction with Subjects experts. This course book provides a fun and practical approach to the subject of mathematics, and at the same time imparting life long skills to the pupils.

The book comprehensively covers the Secondary 1 syllabus as developed by **Ministry of General Education and Instruction.**

Each year comprises of a Student's Book and Teacher's Guide.

The Student's Books provide:

- Full coverage of the national syllabus.
- A strong grounding in the basics of mathematics.
- Clear presentation and explanation of learning points.
- A wide variety of practice exercises, often showing how mathematics can be applied to real-life situations.
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Secondary Mathematics 1

Student's Book

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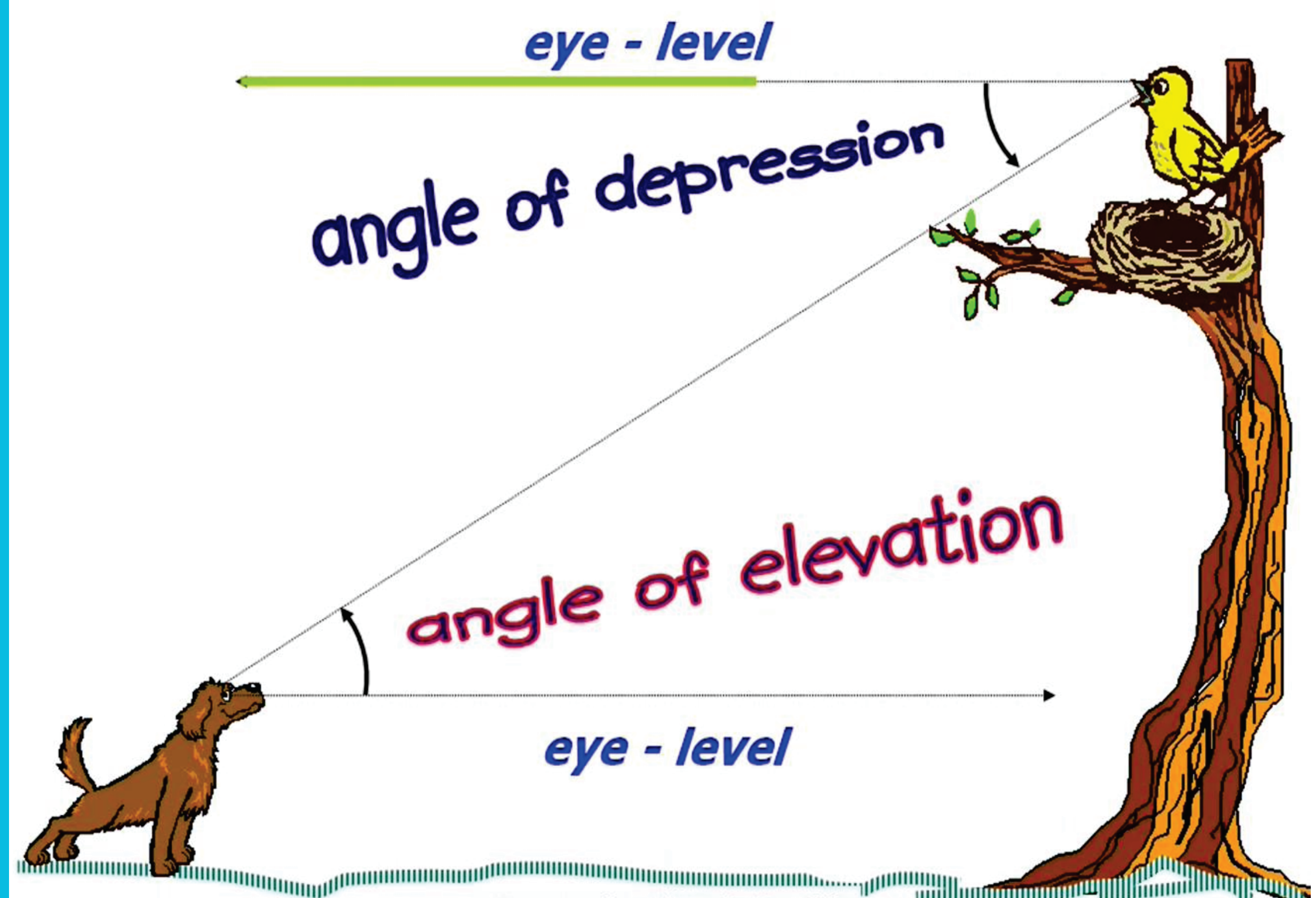

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South Sudan

SECONDARY

1

Mathematics

Student's Book 1

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FOREWORD

I am delighted to present to you this textbook, which is developed by the Ministry of General Education and Instruction based on the new South Sudan National Curriculum. The National Curriculum is a learner-centered curriculum that aims to meet the needs and aspirations of the new nation. In particular, it aims to develop (a) Good citizens; (b) successful lifelong learners; (c) creative, active and productive individuals; and (d) Environmentally responsible members of our society. This textbook, like many others, has been designed to contribute to achievement of these noble aims. It has been revised thoroughly by our Subject Panels, is deemed to be fit for the purpose and has been recommended to me for approval. Therefore, I hereby grant my approval. This textbook shall be used to facilitate learning for learners in all schools of the Republic of South Sudan, except international schools, with effect from 4th February, 2019.

I am deeply grateful to the staff of the Ministry of General Education and Instruction, especially Mr Michael Lopuke Lotyam Longolio, the Undersecretary of the Ministry, the staff of the Curriculum Development Centre, under the supervision of Mr Omot Okony Olok, the Director General for Quality Assurance and Standards, the Subject Panelists, the Curriculum Foundation (UK), under the able leadership of Dr Brian Male, for providing professional guidance throughout the process of the development of National Curriculum and school textbooks for the Republic of South Sudan since 2013. I wish to thank UNICEF South Sudan for managing the project funded by the Global Partnership in Education so well and funding the development of the National Curriculum and the new textbooks. I am equally grateful for the support provided by Mr Tony Calderbank, the former Country Director of the British Council, South Sudan; Sir Richard Arden, Senior Education Advisor of DfID, South Sudan. I thank Longhorn and Mountain Top publishers in Kenya for working closely with the Ministry, the Subject Panels, UNICEF and the Curriculum Foundation UK to write the new textbooks. Finally, I thank the former Ministers of Education, Hon. Joseph Ukel Abango and Hon. Dr John Gai Nyuot Yoh, for supporting me, in my previous role as the Undersecretary of the Ministry, to lead the Technical Committee to develop and complete the consultations on the new National Curriculum Framework by 29 November 2013.

The Ministry of General Education and Instruction, Republic of South Sudan, is most grateful to all these key stakeholders for their overwhelming support to the design and development of this historic South Sudan National Curriculum. This historic reform in South Sudan's education system is intended to benefit the people of South Sudan, especially the children and youth and the future generations. It shall enhance the quality of education in the country to promote peace, justice, liberty and prosperity for all. I urge all Teachers to put this textbook to good use.

May God bless South Sudan. May He help our Teachers to inspire, educate and transform the lives of all the children and youth of South Sudan.

Deng Deng Hoc Yai, (Hon.)
Minister of General Education and Instruction, Republic of South Sudan

Table of Contents

UNIT 1: NATURAL NUMBERS, FACTORS AND PROPORTION

Natural numbers

All over the world, people use numbers. They play a very vital role in our lives and they have been important to human beings for thousands of years.

Exercise 1

In pairs, discuss 5 different ways numbers are useful in our lives. Write these in your exercise book and share with the class.

The Number system

When we write any number, we write it as either one of them or a combination of 10 symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0.

These symbols are called **digits.** When these digits are written together, we form **numerals** that represent numbers. The numbers we use for counting, such as 1, 2, 3, 4, 5… are called **counting numbers** or **natural numbers**.

When we include **zero, 0**, then our set of counting numbers becomes **whole numbers.**

Exercise 2

In pairs, discuss the following questions:

1. Is zero a counting number? Can it be used when counting?
2. What is the largest number we can count?

Natural numbers have place value. The place or position of a digit determines its value.

For example, three thousand four hundred and seventy two is written as 3472 which is really

3000 + 400 + 70 + 2
This means that there 2 ones, 7 tens, 4 hundreds and 3 thousands.

Example

a. Write in numerical form the number "two thousand five hundred and fifty seven"
b. What value does the digit 6 have in the number 1695?

Solution

a. 2557 b. hundreds

Factors of natural numbers

The factors of natural numbers are the natural numbers which divide exactly into it.

Example

The factors of 10 are 1, 2, 5 and 10.

3 is not a factor since 10 ÷ 3 = 3 remainder 1

Even and Odd numbers

A natural number is **even** if it has 2 as one of its factors.
(Or, if it is divisible by 2)
A natural number is **odd** if it is not divisible by 2.

Exercise 3

1. Write the following numbers in numerical form;
   a. Three thousand two hundred.
   b. Twenty two thousand and fifty.
   c. One million, five hundred and fifty two.
2. What is the value of 2 in the numbers in question 1?
3. Write the greatest factor other than itself of :
   a. 12   b. 30   c. 44   d. 56   e. 39
4. What is the least whole number which:
   a. has factors of 2, 3, 5?
   b. has factors 2, 3, 5, 7?
   c. has factors 3, 5, 7?
5. What kind of number do you get when you:
   a. add three even numbers.
   b. add three odd numbers.
   c. multiply an even and an odd number.
   d. add 4 consecutive odd numbers.

Prime numbers

Some numbers have only two factors, one and itself. For example, the factors of 7 are 1 and itself. These kinds of numbers are called **prime numbers.**

Composite numbers

A natural number that has more than two factors is called a composite number. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.

This makes 12 a composite number as well as an even number. 12 can be written as a product of prime factors: 3×2^2

1 is a special number as it is neither prime nor composite.

Example

Express 252 as the product of prime factors

Solution: $252 = 2 \times 2 \times 3 \times 3 \times 7$

$= 2^2 \times 3^2 \times 7$

Exercise 4

In groups, discuss and write the answers in your exercise books

1. a. List all the prime numbers less than 50
   b. Which is the least prime number? What characteristic sets it apart from the others?
2. a. What is the least odd prime number?
   b. Which is the only odd two-digit composite number less than 20?
   c. Which number is a prime number and also a factor of 105, 20 and 30?

3. The two digits of a number are the same. Their product is not a composite number. What is the original number?

Highest common factor

A number which is a factor of two or more other numbers is called a **common factor** of these numbers.

For example, 5 is a common factor of 10 and 15. 7 is a common factor of 14 and 28.

We can use the method of finding prime factors to find the highest common factor for two or more numbers.

Example

Find the highest common factor (HCF) of 12 and 18

Solution: $12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$ Therefore the highest common factor of 12 and 18 is $2 \times 3 = 6$

The highest common factor is also called the **greatest common divisor (GCD).**

Exercise 5

Individually, answer the following questions in your exercise books

1. Find the HCF of :
    a. 9, 12
    b. 24, 32
    c. 108, 144
    d. 25, 50, 75
    e. 22, 33, 44
    f. 10,18,20,36
    g. 32, 56, 72, 88

Lowest common multiple

The multiples of any whole number have that number as a factor. They are found by multiplying the number by 1, then by 2, then by 3 etc.

For example the multiples of 4 are 4, 8, 12, 16, 20, 24, …

The multiples of 6 are 6, 12, 18, 24, ... 12 and 24 are examples of **common multiples** of 4 and 6.

12 is said to be the **lowest common multiple** (LCM) of 4 and 6.

Example

a. Find the common multiples of 10 and 15 less than 70
b. Find the least common multiple (LCM) of 10 and 15

Solution

a. Multiples of 10 less than 70 are 10, 20, 30, 40, 50, 60.

   Multiples of 15 less than 70 are 15, 30, 45, 60.

   The common multiples are 30 and 60.

b. The Lowest Common Multiple (LCM) is 30.

Exercise 6

In pairs, discuss and answer the following question in your exercise book.

1. List the numbers from 1 to 30.
   a. Put a circle around each multiple of 3.
   b. Put a square around each multiple of 4
   c. List the common multiples of 3 and 4 which are less than 30.
2. Find the lowest common multiple of 2, 3, 4 and 5.
3. Find the least natural number that is exactly divisible by 2, 3, 10, 14 and 15.
4. Three bells ring at intervals of 4, 6 and 9 seconds respectively. If they all ring at 8:00 a.m. at what time will they all ring together again?

Fractions

What is a fraction?

If an orange is cut into four equal parts, each part is called a quarter $\left(\frac{1}{4}\right)$.

A fraction is a part of a whole number. E.g. $\frac{1}{4}$ means one part out of the four parts. The top number is called the **numerator** and the bottom number is called the **denominator**.

$$Fraction = \frac{Numerator}{Denominator}$$

Types of fractions

a. Proper fractions

$\frac{1}{2}, \frac{3}{4}, \frac{7}{10}$ have something in common. What is it?

The numerator is smaller than the denominator. This kind of fraction is called a **proper fraction**.

b. If the numerator is bigger than the denominator then the fraction is called **an improper fraction**, for example, $\frac{5}{2}, \frac{3}{2}, \frac{10}{7}$.

c. We can have a whole number and a fraction, for example one and a half oranges. We write this as $1\frac{1}{2}$. This is called a **mixed number** because it consists of a whole number and a fraction.

We can change from improper fractions to mixed numbers and vice versa.

Example

1. Change $1\frac{1}{6}$ into an improper fraction.

Solution: $$1\frac{1}{6} = \frac{(1\times 6)+1}{6} = \frac{7}{6}$$

2. Change $\frac{5}{3}$ into a mixed number.

Solution: $$\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

3. Express $8\frac{3}{4}$ as an improper fraction.

Solution: $$8\frac{3}{4} = \frac{(8\times 4)+3}{4} = \frac{35}{4}$$

4. Express $\frac{16}{5}$ as a mixed number.

Solution: $$\frac{16}{5} = 3\frac{1}{5}$$

Comparing fractions

Fractions can only be compared when their denominators are the same.

Example

Arrange $\frac{2}{3},\frac{4}{5},\frac{3}{4}$ in ascending order

Find the LCM of 3, 5 and 4

LCM=60

$$\frac{2}{3}=\frac{2\times 20}{3\times 20}=\frac{40}{60},\ \frac{4}{5}=\frac{4\times 12}{5\times 12}=\frac{48}{60},\ \frac{3}{4}=\frac{3\times 15}{4\times 15}=\frac{45}{60}$$

Arranging the fraction in ascending order (least to greatest), we get

$$\frac{40}{60},\frac{45}{60},\frac{48}{60}=\frac{2}{3},\frac{3}{4},\frac{4}{5}$$

Exercise 7

1. Write each of the following in numerals:
   a. Two-thirds
   b. Three-fifths
   c. Six-sevenths
   d. Three-tenths
   e. Nine-hundredths
2. Find the value of:
   a) two-thirds of 1 hour
   b) three-fifths of 1kg
   c) one-quarter of 1 ton
3. Copy and insert the missing numbers:

   a) $\frac{2}{5}=\frac{6}{\ }$ c) $\frac{35}{\ }=\frac{5}{6}$

   b) c) $\frac{40}{\ }=\frac{\ }{15}=\frac{2}{3}$ d) $\frac{18}{90}=\frac{\ }{30}=\frac{3}{\ }=\frac{\ }{5}$

4. Write three fractions equivalent to:

a) $\frac{3}{7}$ b) $\frac{2}{5}$

5. Arrange the following fraction in ascending order;

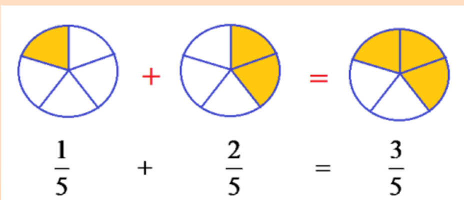
a) $\frac{2}{9}, \frac{4}{5}, \frac{2}{3}$ b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{10}, \frac{4}{15}$

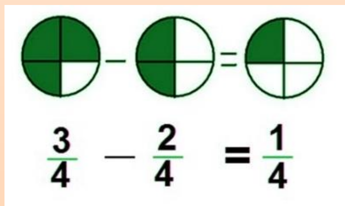
Working with fractions

a. Addition and subtraction of fractions

Fractions can be added or subtracted only when their denominators are equal.

For example, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ and $\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$, only add or subtract the numerator.





$$\frac{7}{8}-\frac{3}{8}=\frac{7-3}{8}=\frac{\cancel{4}^{1}}{\cancel{8}_{2}}=\frac{1}{2}$$

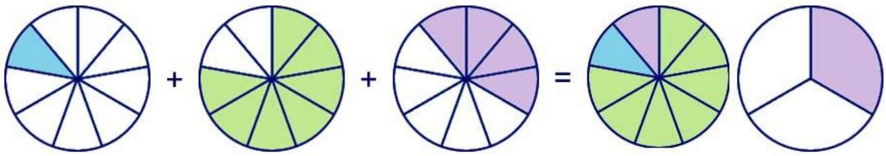
Fractions should always be cancelled down to their lowest terms.

We can show this calculation in a diagram:

$$\frac{1}{9}+\frac{7}{9}+\frac{4}{9}=\frac{1+7+4}{9}=\frac{12}{9}=1\frac{3}{9}=1\frac{1}{3}$$

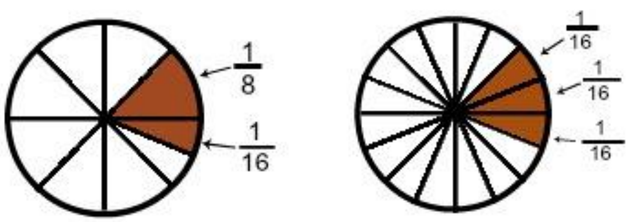
Top-heavy or **improper fractions** should be written as mixed numbers.

Again, we can show this calculation in a diagram:



If the denominators are not the same, then we have to make them the same first by getting the LCM of the numbers, before we add or subtract

$$\frac{1}{3}+\frac{2}{5}=\frac{5+6}{15}=\frac{11}{15} \text{ and } \frac{5}{6}-\frac{1}{5}=\frac{25-6}{30}=\frac{19}{30}$$



$$\frac{1}{8}+\frac{1}{16}=\frac{2}{16}+\frac{1}{16}=\frac{3}{16}$$

Activity

In pairs, discuss the following examples and then attempt the questions that follow. The teacher should make sure that the students show understanding by choosing 3 students to explain to the whole class.

Evaluate a) $6\frac{3}{4}+2\frac{1}{3}$

b) $4\frac{4}{5} - 2\frac{1}{4}$

c) $4\frac{1}{10} + 2\frac{1}{5} - 2\frac{2}{3}$

Solutions

a) $6\frac{3}{4} + 2\frac{1}{3} = 8\frac{9+4}{12} = 8\frac{13}{12} = 9\frac{1}{12}$

- Add the whole numbers
- Find the LCM of 4 and 3 and add the corresponding numerators
- Change the improper fraction $\frac{13}{12}$ into $1\frac{1}{12}$ and add the whole number 1 to 8.

b) $4\frac{4}{5} - 2\frac{1}{4} = 2\frac{16-5}{20} = 2\frac{11}{20}$

- Subtract the whole numbers
- Find the LCM of the denominators, 5 & 4
- Find the corresponding numerators and subtract.

c) $4\frac{1}{10} + 2\frac{1}{5} - 2\frac{2}{3} = 4\frac{3+6-20}{30} = 4 - \frac{11}{30} = 3\frac{19}{30}$

- Add & subtract whole numbers: 4+2-2=4
- Find LCM of 10, 5 & 3 and find corresponding numerators
- Add and subtract.

Exercise 9

Evaluate and simplify:

1. $6\frac{1}{2} + 3\frac{3}{4}$

2. $2\frac{7}{8} + 1\frac{4}{5}$

3. $4\frac{1}{5}+2\frac{3}{10}$

4. $5\frac{3}{4}-2\frac{7}{8}$

5. $3\frac{1}{2}-2\frac{3}{4}$

6. $10\frac{7}{16}-3\frac{3}{8}$

7. $-\frac{2}{3}-\frac{4}{5}+\frac{3}{10}$

8. $3\frac{1}{6}-2\frac{1}{3}+\frac{7}{12}$

9. $-2\frac{6}{13}+1\frac{1}{2}$

10. $4\frac{1}{6}+1\frac{11}{20}-3\frac{7}{15}$

11. $10\frac{1}{2}-3\frac{2}{3}+4\frac{1}{5}+2\frac{3}{4}$

Multiplication of fractions

To multiply two fractions, we multiply the two numerators to get the new numerator and multiply the two denominators to get the new denominator.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

Example

a) $\frac{3}{5}\times\frac{8}{7}$ b) $\frac{2}{3}\times\frac{6}{11}$ c) $2\frac{7}{10}\times1\frac{2}{3}$

Solutions

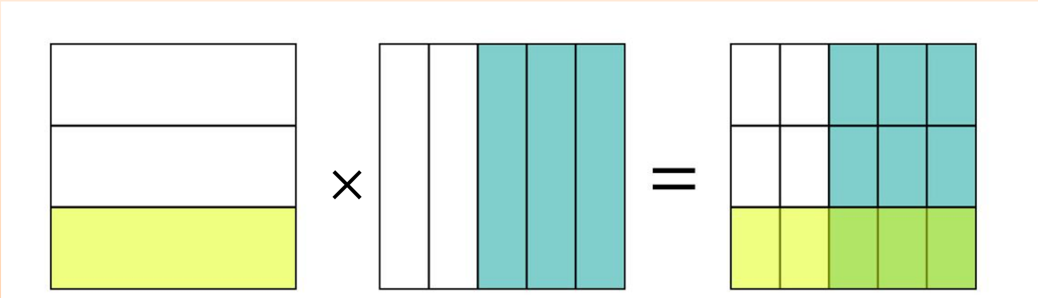
a) $\frac{3}{5}\times\frac{8}{7}=\frac{24}{35}$

b) $\frac{2}{3}\times\frac{6}{11}=\frac{12}{33}=\frac{4}{11}$

To help make multiplication easier, we can cancel any common factors in the numerator and denominator before multiplying: $\frac{2}{3}\times\frac{6}{11}=\frac{2}{1}\times\frac{2}{11}=\frac{4}{11}$

c) $2\frac{7}{10}\times1\frac{2}{3}=\frac{27}{10}\times\frac{5}{3}=\frac{9}{2}=4\frac{1}{2}$

Mixed numbers should be first changed to improper fractions before multiplying.



$$\frac{1}{3} \times \frac{3}{5} = \frac{3}{15} = \frac{1}{5}$$

Exercise 10

Working in groups evaluate the following showing all your working clearly and simplifying where possible

1. Find:

a. $\frac{2}{5} \times \frac{3}{4}$

b. $-\frac{5}{4} \times \frac{2}{3}$

c. $\frac{4}{27} \times \frac{21}{16}$

d. $1\frac{1}{2} \times \frac{1}{3}$

2. Find:

a. $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$

b. $\frac{3}{8} \times \left(-\frac{2}{3}\right) \times \frac{3}{5}$

c. $\frac{2}{5} \times 4 \times 1\frac{5}{8}$

d. $\frac{2}{3} + \frac{3}{4} \times \frac{2}{3}$

e. $\frac{3}{5} - \frac{1}{3} \times \frac{6}{7}$

f. $\frac{2}{3} + \frac{3}{4} \times \frac{2}{3} - \frac{1}{2}$

Reciprocals

Task

See if you can solve these problems in your head by multiplying the first number by the reciprocal of the second number:

1. $8 \div \frac{1}{5}$
2. $10 \div \frac{1}{10}$
3. $3 \div \frac{3}{8}$

(*Hint:* All the answers will be whole numbers.)

Clearly, $3 \times \frac{1}{3} = 1$

$\frac{1}{3}$ is called the reciprocal of 3.

3 is called the reciprocal of $\frac{1}{3}$.

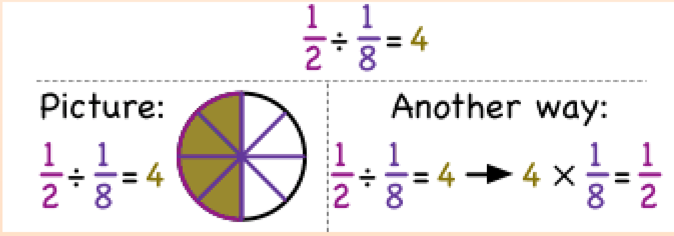
One number is the **reciprocal** of another if their product is 1.
The reciprocal of a fraction is obtained by interchanging the numerator and the denominator, i.e. by inverting the fraction.

$$\frac{a}{b} \times \frac{b}{a} = 1$$

Therefore $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$

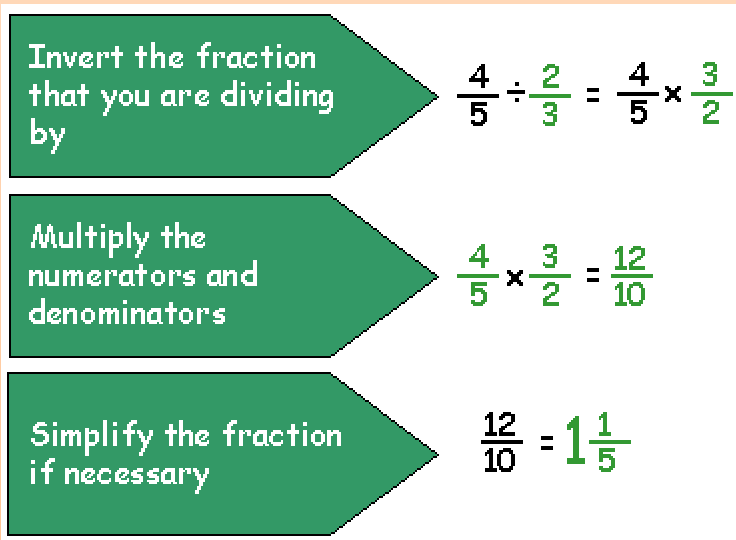
For example, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$ since $\frac{5}{6} \times \frac{6}{5} = 1$.

c. Dividing fractions



$$\frac{1}{2} \div \frac{1}{8} = \frac{\frac{1}{2}}{\frac{1}{8}} = \frac{\frac{1}{2} \times 8}{\frac{1}{8} \times 8} = 4$$

To divide fractions take the reciprocal (invert the fraction) of the divisor and multiply the dividend.



This is the quickest technique for dividing fractions. The top and bottom are being multiplied by the same number and, since that number is the reciprocal of the bottom part, the bottom becomes one.

How to Make a Fraction on a Scientific Calculator

By default, scientific calculators, like regular ones, display fractions as decimals. So if you enter a simple fraction, such as $\frac{1}{2}$, the display reads 0.5. Some – but not all – scientific calculators offer a feature that allows you to display fractions without making the conversion. Using this feature, you can enter a complex fraction and simplify it right on your calculator. Calculators with this feature also allow you to enter a number composed of an integer and a fraction, such as $1\frac{1}{4}$. If your calculator doesn't have this feature, you can use a workaround to manipulate fractions.

Exercise 11

In pairs, discuss how you would divide the following fractions. Use a calculator.

a. $\frac{2}{3} \div \frac{1}{6}$

b. $1\frac{1}{3} \div 3\frac{1}{2}$

c. $2\frac{3}{4} \div \frac{2}{3}$

d. $\frac{4}{5} \div 3$

e. $\dfrac{2 + \frac{1}{3}}{2 - \frac{1}{3}}$

f. $\dfrac{\frac{2}{3} + \frac{3}{4}}{\frac{2}{3} - \frac{3}{4}}$

g. $\dfrac{\frac{1}{3} + \frac{1}{4}}{1 - \frac{1}{5}}$

Fractions, percentages and decimals

Changing fractions to decimals

You can convert all fractions to decimals. The decimal forms of rational numbers either end or repeat a pattern. To convert fractions to decimals you just divide the top by the bottom — divide the numerator by the denominator — and if the division is not exact, you can stop after a certain number of decimal places and either round or truncate.

Some fractions can be expressed as exact decimals.

$$\frac{1}{2} = \frac{5}{10} = 0.5 \qquad \frac{1}{4} = \frac{25}{100} = 0.25$$

$$\frac{3}{5} = \frac{6}{10} = 0.6 \qquad \frac{7}{10} = 0.7$$

Recurring decimals

Some fractions cannot be expressed as exact decimals.

$\frac{2}{3} = 0.6666\ldots$ (6 is repeated) $= 0.66$ to 2d.p. (truncated) $= 0.67$ to 2d.p. (rounded)

$\frac{5}{6} = 0.8333\ldots$ (3 is repeated) $= 0.83$ to 2d.p. (truncated) $= 0.83$ to 2d.p. (rounded)

$\frac{1}{7} = 0.142857142857\ldots$ (142857 is repeated) $= 0.143$ to 3d.p. (rounded)

Decimals in which one or more figures are continually repeated in the same order are called **recurring or repeating decimals**. They are written by placing a dot over the recurring figure, or dots on the first and last digits of the repeating block of digits.

$$\frac{2}{3} = 0.\dot{6} \qquad \frac{5}{6} = 0.8\dot{3} \qquad \frac{1}{7} = 0.\dot{1}4285\dot{7}$$

Changing recurring decimals into fractions:

Example

a) $0.\dot{6}$

b) $0.8\dot{3}$

Solution

a) Let $r = 0.66666\ldots$

$10r = 6.66666\ldots$

- Multiply by 10 because the recurring number is in the **tenths** position.
- Subtract r from $10r$ to get rid of the decimal part of the number
- We are then able to find r as a fraction

$10r - r = 9r$

$6.6666... - 0.6666... = 6$

Therefore, $9r = 6$ hence $r = \frac{6}{9} = \frac{2}{3}$

b) Let $r = 0.833333...$

$10r = 8.33333...$

$100r = 83.33333...$

$100r - 10r = 90r$

$83.3333... - 8.3333... = 75$

$90r = 75$ hence $r = \frac{75}{90} = \frac{5}{6}$

- Multiply by 100 because the recurring number is in the **hundredths** position
- To get rid of the decimal part of the number, subtract *10r* from *100r*
- We are then able to find *r* as a fraction

Changing decimals to fractions

All terminating and recurring decimals can be expressed as fractions.

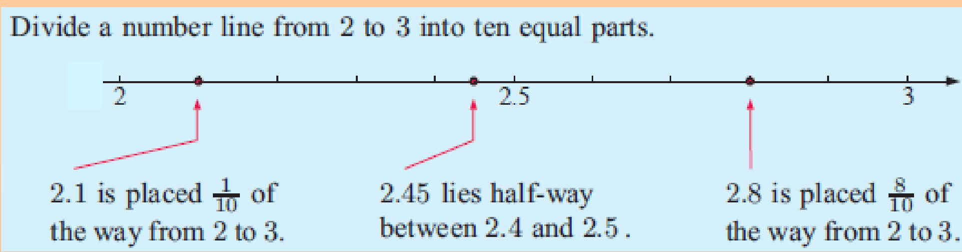
$0.5 = \frac{5}{10} = \frac{1}{2}$ $0.32 = \frac{32}{100} = \frac{8}{25}$

Decimal places

Ordering Decimal Numbers

Just like whole numbers, decimal numbers can be shown on a number line.

For example: 2.1, 2.45 and 2.8 can be placed on number line:



2.1 is the least while 2.8 is the greatest.

A number with lots of decimal digits may be less than a number with fewer decimal digits e.g. 1.4 > 1.32568

Rounding

What is 1.32568 to 3 decimal places?

The numbers **with 3 decimal places** closest to 1.32568 are **1.325 and 1.326**

As 1.32568 is closer to 1.326, 1.32568 = 1.326 to 3d.p.

What is 1.5 to the nearest whole number?

1.5 is exactly half way between 1 and 2 the convention is to round up to 2.

Exercise 12

1. In a closing-down sale, a shop offers 50% off the original prices. What fraction is taken off the prices?
2. In a survey, one in five people said they preferred a particular brand of milk. What is this figure as a percentage?
3. Keji is working out a problem involving $\frac{1}{4}$. She needs to enter this into a calculator. How would she enter $\frac{1}{4}$ as a decimal on the calculator?
4. Mr. Achan pays tax at the rate of 25% on his income. What fraction is Mr. Achan's income in this?
5. When Deng was buying his plot of land he had to put down a deposit of $\frac{1}{10}$ of the value of the piece of land. What percentage was this?
6. I bought a coat in the December sales with $\frac{1}{3}$ off the original price. What percentage was taken off the price of the coat?
7. Mary bought some fabric that was 1.75 metres long. How could this be written as a fraction?

Standard form

Astronomers, biologists, engineers, physicists and many others encounter quantities whose measures involve very small or very large numbers. For example, the distance of the Earth from the Sun is approximately 144 000 000 000 metres and the distance that light will travel in 1 year is 5 870 000 000 000 metres.

It is sometimes tedious to write or work with such numbers. This difficulty is overcome by writing such numbers in standard form.

E.g. $144\,000\,000\,000 = 1.44 \times 10^{11}$

$5\,870\,000\,000\,000 = 5.87 \times 10^{12}$

If a quantity is written as the product of a power of 10 and a number that is greater than or equal to 1 and less than 10, then the quantity is said to be expressed in **standard form** (or **scientific notation**).

For example, $658 = 6.58 \times 10^2$

Note:

- We have expressed 658 as a product of 6.58 and a power of 10. Clearly, 6.58 is between 1 and 10. So the standard form of 658 is 6.58×10^2.
- The digit 6 in 658 is in the hundreds (10^2) place

In general:

In converting a number to standard form, the place value of the first digit of the number is the power of ten.

E.g. $56 = 5.6 \times 10^1$

$564 = 5.64 \times 10^2$

$5648 = 5.648 \times 10^3$

$0.05648 = 5.648 \times 10^{-2}$

Note:

5 is in the hundredths (10^{-2}) place.

Exercise 13

In pairs, convert these numbers into standard form.

1. 0.000 000 000 467
2. 0.000 000 031 6
3. 547
4. 0.000 000 005 09
5. 4 660 000 000
6. 11 000 000
7. 0.000 703
8. 2 490 000
9. 0.0768
10. 54 400

Percentages

We use percentages to compare a portion with a whole amount of 100%.

Percentages are used to describe interest rates, changes in profit levels, test results, inflation and much more.

All fractions and decimals may be converted into percentage form by first writing them as a fraction with a denominator of 100.

Exercise 14

1. **Write these decimals as fractions:**

   a) 0.3
   b) 0.5
   c) 0.6
   d) 0.02
   e) 0.05
   f) 0.25
   g) 0.36
   h) 0.125

2. **Write these fractions as decimals:**

   a) $\frac{7}{10}$
   b) $\frac{1}{5}$
   c) $\frac{2}{5}$
   d) $\frac{3}{4}$
   e) $\frac{7}{8}$
   f) $\frac{2}{3}$
   g) $\frac{9}{20}$

h) $\frac{7}{25}$

3. Write these percentages as decimals:

a) 3%
b) 30%
c) 25%
d) 80%
e) 8%
f) 12%
g) 67%
h) 17.5%

4. Write these percentages as fractions:

a) 20%
b) 75%
c) 5%
d) 30%
e) 40%
f) 15%
g) 24%
h) 35%

5. Write these decimals as percentages:

a) 0.25
b) 0.5
c) 0.7
d) 0.07
e) 0.45
f) 0.09
g) 0.4
h) 0.375

6. Write these fractions as percentages:

a) $\frac{1}{10}$
b) $\frac{1}{5}$
c) $\frac{9}{10}$
d) $\frac{3}{4}$
e) $\frac{4}{5}$
f) $\frac{17}{20}$
g) $\frac{1}{3}$
h) $\frac{2}{3}$

At the end of the exercise:

1. Explain how to convert...
   - a decimal to a fraction
   - a fraction to a decimal
   - a percentage to a fraction or decimal

- a fraction or decimal to a percentage.

2. Give some examples of real contexts where...
   - decimals are used
   - fractions are used
   - percentages are used.

Profit and loss

Profit is realized when the selling price is higher than the buying price.

Example 1

John bought a basin for SSP 172. He later sold it for 208 pounds. What profit did he make?

Solution

Selling price = SSP 208.

Buying price = SSP 172.

Profit = SSP (208-172).

= SSP 36.

Loss is realized when the buying price is higher than the selling price.

Example 2

Roselyne bought a radio for SSP 720. She later sold it at SSP 630. What loss did she make?

Solution

Buying price = SSP 720.

Selling price = SSP 630.

Loss = SSP (720-630)

= SSP 90

Calculating Percentage Profit and Loss

Cost price The amount for which an article is bought is called its cost price.

Selling price The amount for which an article is sold is called its selling price.

Profit or gain When selling price is greater than cost price then there is a gain.

Profit or gain = selling price – cost price

Loss When selling price is less than cost price then there is a loss.

Loss = cost price – selling price

Notes: The percentage profit or loss is always found on the cost price

Calculating Percentage Profit and Loss

I. Percentage Profit $= \frac{profit}{cost\ price} \times 100$

II. Percentage Loss $= \frac{loss}{cost\ price} \times 100$

Task

Find the percentage profit or loss in the examples above.

Percentage profit and loss including overheads

Sometimes after purchasing an article, we have to pay more money for things like transportation, repair charges, local taxes. These extra expenses are called **overheads**. For calculating the total cost price, we add overheads to the purchase price.

1. Kiden purchased an old scooter for SSP 12 000 and spent SSP 2850 on its overhaul. Then, she sold it to her friend Ohide for SSP 13 860. What was her percentage loss or gain when selling to her friend?

   Solution:

   Cost Price of the scooter = SSP 12 000, overheads = SSP 2850.

Total Cost Price = SSP (12 000 + 2850) = SSP 14 850.

Selling price = SSP 13 860.

Since Selling Price < Cost Price, Kiden makes a loss.

Loss = SSP (14 850 – 13 860) = SSP 990.

Percentage Loss = $\frac{loss}{total\ Cost\ price} \times 100\ \%$

$= \frac{990}{14850} \times 100\ \%$

$= 6\%$

2. A vendor bought oranges at 20 for SSP 56 and sold them at SSP 35 per dozen. Find his percentage gain or loss.

Solution:

Cost Price of 20 oranges = SSP 56

Cost Price of 1 orange = SSP $\left(\frac{56}{20}\right)$ =SSP 2.8

Selling Price of 12 oranges = SSP 35

Selling Price of 1 orange = SSP $\left(\frac{35}{12}\right)$ = SSP $2.91\dot{6}$

Since, Selling Price > Cost Price, the vendor gains.

Gain = SSP $(2.91\dot{6} - 2.8)$ = SSP $0.11\dot{6}$.

Percentage Gain = $\left(\frac{profit}{Cost\ Price}\right) \times 100\ \%$

$= \left(\frac{0.11\dot{6}}{2.8}\right) \times 100\%$

$= 4\frac{1}{6}\ \%$

Exercise 15: Work in pairs.

1. A trader bought 8 trays of eggs at 240 pounds per tray, eight eggs broke and he sold the rest at 12 pounds per egg. If a tray holds 30 eggs, how much money did he get as profit?

2. Jacob bought 250 chicken whose average mass was $1\frac{1}{2}$kg. The buying price per kilogram was 150pounds. He then sold each chicken for 300 pounds, what profit did Jacob make?

3. Jacinta bought 15 bags of fruits at 450 pounds per bag. She spent 500 pounds on transport, $1\frac{1}{2}$ bags of the fruits got spoilt and she sold the rest at 400 pounds per bag. What was her loss?

4. Joel bought 5 pairs of shoes at 250 pounds per pair. He later sold them. If the buyer gave six two hundred Sudanese pound notes and Joel gave him back 105 pounds. How much was the loss?

Exercise 16

In groups, discuss the following questions.

1. The price of milk is raised from 50 pounds to 55 pounds per litre.
   a. Juma buys 5 litres every day. How much more money is he spending every day because of this price increase?
   b. Mary has a shop and buys milk to sell. How much was she spending before the price increase if she buys 10 litres every day?
   c. She sells her milk at 55 pounds per litre, what was her percentage profit?
   d. After the price increase, she decides to sell her milk at 65 pounds. What is her percentage profit now?

Project work (work in groups)

Your group wants to start a kiosk near your village.

a. What are the fast moving goods that you can stock your kiosk with? Start with at least 5.
b. How much money does your group need in order to buy your first stock?
c. How much stock will buy of each of the goods?
d. How much will sell each good?
e. Keep records of how much money is spent every day as well as how much money you have at the end of every day. Do this for 2 weeks. Have you made a profit or loss? How much per week?
f. At the beginning of week 3, you decide to increase your stock of every item by 10%. How much profit/loss do you make at the end of that week?
g. You then decide to close the kiosk after week and you split the profits made amongst yourselves. How much does each member get? What percentage increase/decrease on their contribution does each member have?

UNIT 2: MEASUREMENT

Measurements of length, area, volume, and capacity, mass and time enable us to answer every day questions such as:

- how far school is from home.
- how big the school compound is.
- how many kilograms of sugar your family uses every month.

In this unit we use **the metric system** of measurement developed in France in 1789. In this system, the **metre** is the standard unit of length and the **kilogram** is the standard unit for mass.

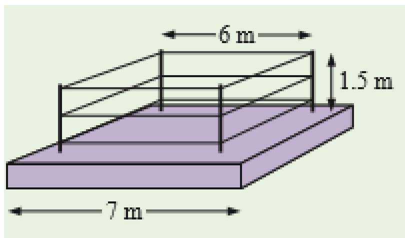
Discussion question

A boxing ring has the dimensions shown in the diagram. There are 3 ropes on each side of the ring, and each rope is 6 m long. The ropes are connected to the corner posts which are 1.5 m high.

The ring is 7 m long on all sides.

Find:

- The total length of the ropes.
- The total length of the corner posts.



The Metre

The metre (m) is the standard unit for length in the metric system. From the metre, other lengths were devised to measure shorter and longer distances:

1 kilometre (km) = 1000 metres (m)

1 metre (m) = 100 centimetres (cm)

1 centimetre (cm) = 10 millimetres (mm)

Exercise 1

In groups, discuss the following:

1. What unit of length would you use to measure:
   - The length of a paper clip?
   - The length of a journey by train?

- The length of an ant?
- The height of a flag pole?

2. What would the correct estimate of the following be:
   - The width of a road – 3 cm, 3 m, 30 m, 300 mm?
   - The length of a new pencil – 6 mm, 6 cm, 6 km, 6 m?
   - The distance from Nairobi to Juba - 8000 km, 8000 m, 8000cm, 8000 mm

Conversion of units

Often we need to convert from one unit to another. **When we are converting from a larger unit to a smaller unit, we must multiply.**

For example, 2 m = (2 × 100) cm= 200 cm

When we are converting from a smaller unit to a larger unit, we must divide.

For example, 2000 m = (2000 ÷ 1000) km = 2 km

Exercise 2

1. Express the following in centimetres.

   a) 8 m b) 16.4 m c) 40 mm d) 6000m e) 5 km

2. Express the following in metres.

   a) 5 km b) 2000km c) 600 cm d) 36 500 mm e) 600 cm

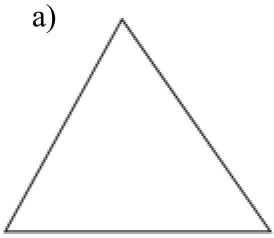
Perimeter

The perimeter of a closed figure is the total length of the boundary of the figure. **The perimeter of a polygon is found by adding the lengths of all the sides.**

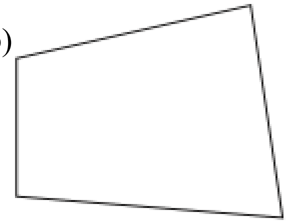
Exercise 3

1. Measure with your ruler the lengths of the sides of the figures below. Hence find the perimeter of each figure.

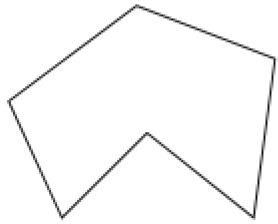
a)



b)

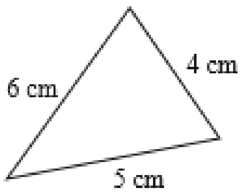


c)

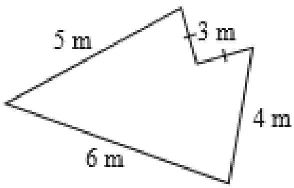


2. Find the perimeter of each of the following figures:

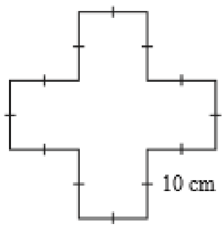
a)



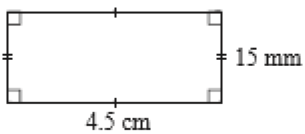
b)



c)



d)



Circumference of a circle

The **diameter** of a circle is the distance across the circle measured through its centre. The **circumference** of a circle is the perimeter or length around its boundary. The common distance from the center of the circle to its points is called **radius**.

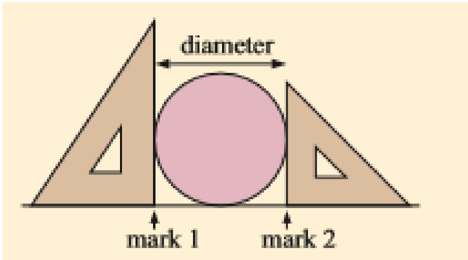
Task 1: To be completed in groups.

You will need:

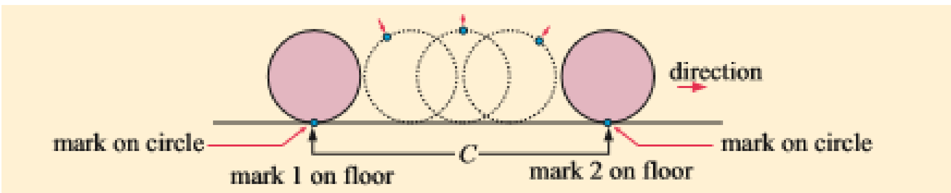
- Cylindrical objects such as water bottle, toilet roll or piece of pipe.

- Two set squares, a ruler and a pencil

Step 1: Measure the distance across the object using the two set squares as shown below. The distance between the two marks is the diameter of your object.



Step 2: Mark a point on the circumference of your object and then roll it along a flat surface for one complete revolution as shown. The distance between the two marks is the circumference of your object.



Step 3: Copy the table below and fill it in with measurements from your objects

Circular object	*Circumference*	*Diameter*

What do you notice?

Step 4: Compare your results with those of the other groups.

Whatever the object, the circumference is roughly three times the diameter.

$$Circumference \approx 3 \times diameter\ .$$

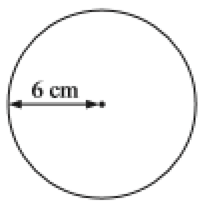
In fact, the ratio $Circumference : diameter = \pi$, a constant that cannot be written exactly as a decimal, $\pi = 3.141592654...$ Wherever possible you should use your calculator π button.

$$Circumference = \pi \times diameter$$

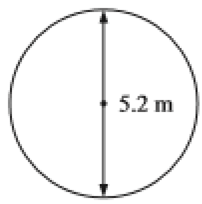
Exercise 4

1. Find the circumference (to 2 decimal places):

a)



b)

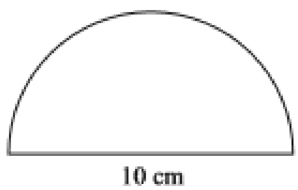


2. A circular flower bed has a radius of 2.5 m. Find the perimeter of the flower bed.

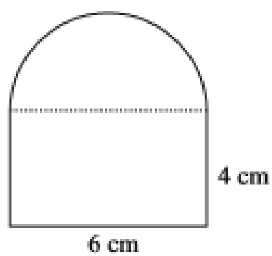
3. A bicycle wheel has radius 40 cm.
   a) Find the circumference of the wheel.
   b) How many kilometres would be travelled if the wheel rotates 10 000 times?
   c) How many times does the wheel rotate if the bicycle is ridden 10 km?

4. Find the perimeter, correct to 2 decimal places of:

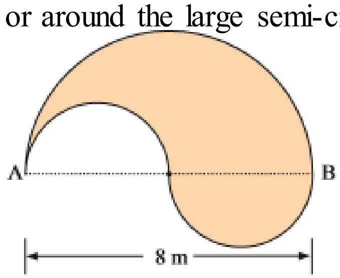
a)



b)



5. Consider the farm below. Which is the shorter path from A to B: around the 2 smaller semi-circles or around the large semi-circle?

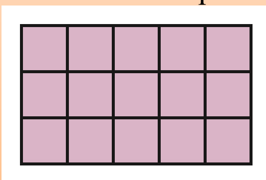


Area

Area is the amount of surface inside a region.

The area of a closed figure is the number of square units it encloses.

For example, the rectangle shown below has an area of 15 square units because it has 3 rows of 5 squares or 5 columns of 3 squares ($3\times5=5\times3=15$ squares). If each square was 1 m by 1m, the area would be 15 square metres. If each square was 1 cm by 1 cm, the area would be 15 square centimetres.



$$Area\ of\ a\ rectangle = length \times width$$

Area of Polygons

Task 2: Finding the areas of different shapes

In groups of 4

1. Draw different sizes of the following polygons on squared paper and cut them out
   i) Triangle
   ii) Parallelogram
   iii) Trapezium
   iv) Kite
2. Count the squares inside each shape to estimate the area
3. Cut up each shape to form a rectangle and calculate the area of the rectangle.
4. Derive general formulae for calculating the area of these different shapes.

Task 3: Length and Area

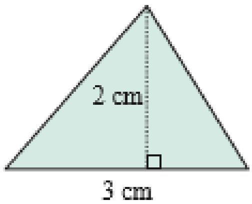
In groups,

- Estimate the length and width of the floor of your classroom. Give reasons for your answer.
- Explain how you would measure the length and width of your classroom.
- Measure and give your measurements in cm and m.
- Hence find the area of the classroom in cm^2 and m^2.
- How do we convert from cm^2 to m^2 and from m^2 to cm^2?

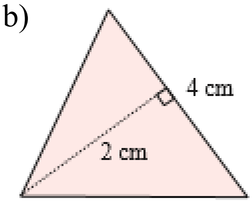
Exercise 5

1. Find the areas of the shapes.

a)



b)



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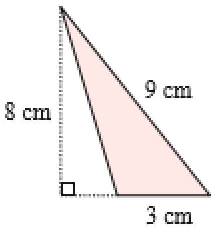
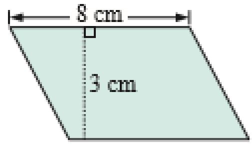
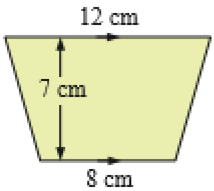


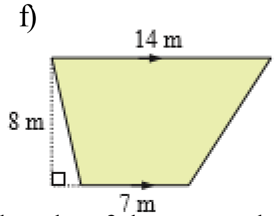
d)



e)



f)

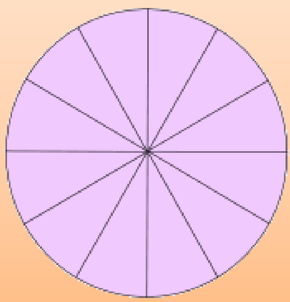


2. A rectangle is 12cm by 8 cm. If the length of the rectangle is increased by 4cm, by how much must the width be changed so that the area remains the same?

Area of a circle:

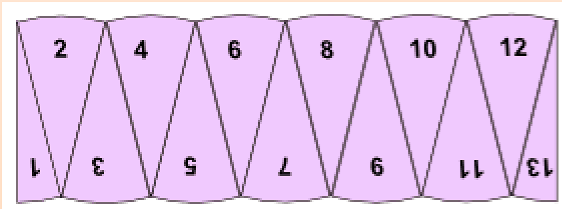
Task: Finding the area of a circle. Work in groups.

Here is a way to find the formula for the area of a circle:

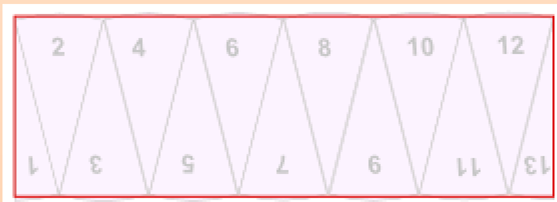


Cut a circle into equal sectors (12 in this example)

Rearrange the 12 sectors like this:

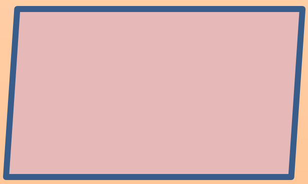


Which resembles a parallelogram:



- What are the (approximate) height and width of the parallelogram?
- What would happen to the parallelogram if the circle was divided into more equal sectors?

The more we divide the circle up, the closer we get to a parallelogram, with length half the Circumference and perpendicular height the radius.



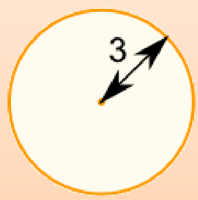
radius

$$\frac{1}{2}C = \boldsymbol{\pi} \times \text{radius}$$

$$\text{Area} = (\boldsymbol{\pi} \times \text{radius}) \times (\text{radius})$$

$$= \boldsymbol{\pi} \times \text{radius}^2$$

Example: What is the area of a circle with radius of 3 m?



Radius = r = 3

Area= πr^2

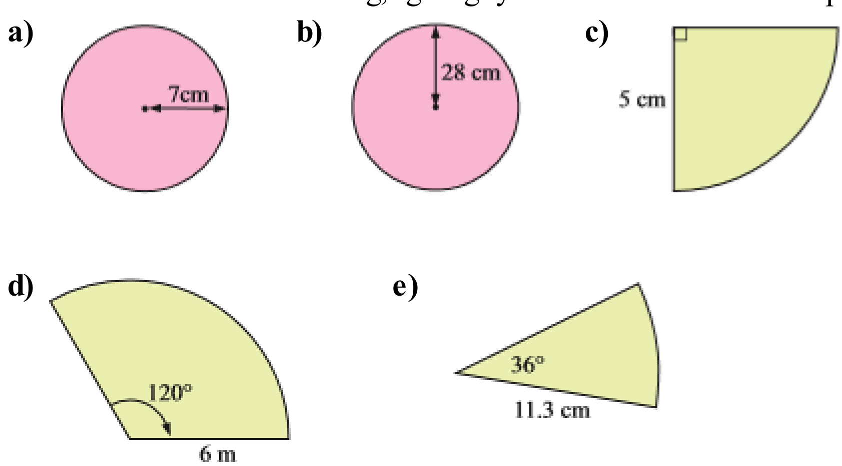
$= \pi \times 3^2$

$= 3.14159... \times (3 \times 3)$

$=$ **28.27 m²** (to 2 decimal places)

Exercise 6: To be done in pairs

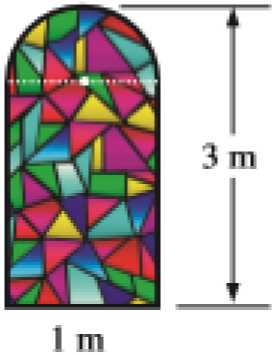
3. Find the area of the following, giving your answer to 2 decimal places



4. A circle has a diameter of 2.6 cm. Find, correct to 2 decimal places
   a) its perimeter b) its area

5. A goat is tied to a post by a rope 5.4 m long. What maximum area can the goat graze?

6. The illustration shows the dimensions of a stained glass window. Find the area of the window



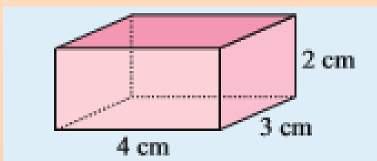
Surface area

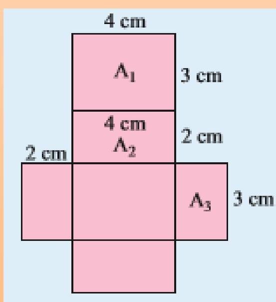
The **surface area** of a three dimensional figure with plane faces is **the sum of the areas of the faces.** The surface area of a solid will equal the area of the net which forms it. It is therefore helpful to draw the net first.

Software that demonstrates nets can be found at http://www.peda.com/poly/

Example:

Find the total surface area of the rectangular box below.





The net of the box is shown:

Area of $A_1 = 4 \times 3 = 12cm^2$

Area of $A_2 = 4 \times 2 = 8\ cm^2$

Area of $A_3 = 2 \times 3 = 6cm^2$

Total surface area

$2 \times A_1 + 2 \times A_2 + 2 \times A_3 = 52cm^2$

Task 3

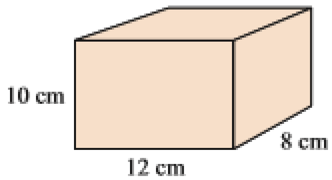
Find the length, width and height of your classroom in metres. Calculate the total surface area of your classroom in m^2.

Exercise 7

1. Find the total surface area of a cube with sides 3 cm

2. Find the surface area of the following rectangular prisms:

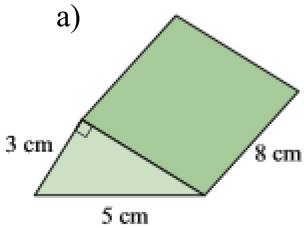
a)



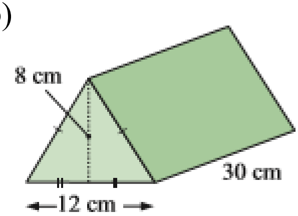
b)

3. Find the surface area of the following triangular prisms:

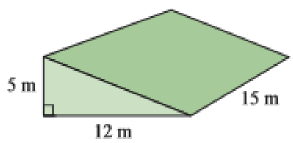
a)



b)

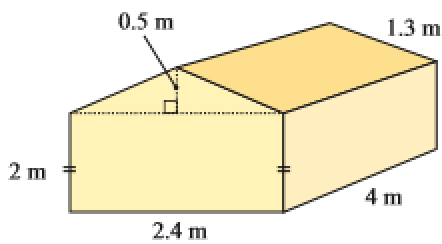


c)



4. A room has dimensions 4 m by 3 m by 2.4 m high. Find the cost of painting the inside of the room (walls & ceiling) if 1 litre of paint costs SSP13.20 and each litre covers 5 m^2.

5. A tent made from canvas is shown below. Find the total cost of the canvas if it costs SSP12.80 per square metre. (the tent's floor is also made from canvas)

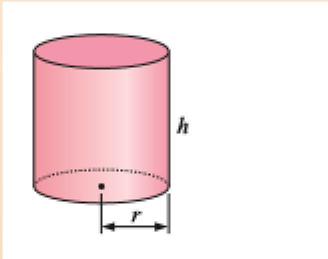


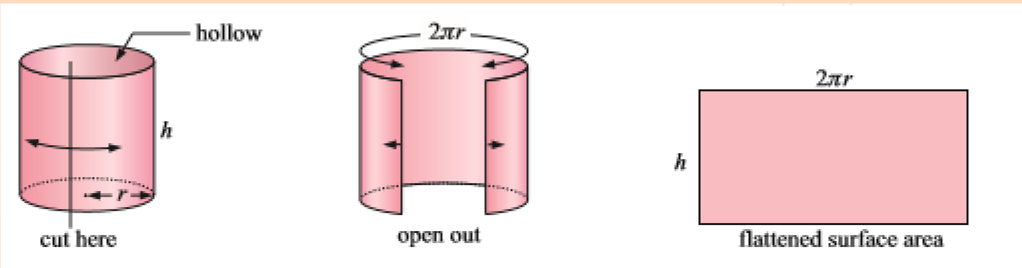
Surface area of solids with curved surfaces

We will consider the outer surface area of cylinders and spheres.

Cylinders

Consider the cylinder shown alongside. If it is cut, opened out and flattened onto a plane surface, it takes the shape of a rectangle.





You could verify this by taking the label of a cylindrical can e.g. a water bottle or soda can. The length of the rectangle is the same as the circumference of the cylinder.

So the outer surface area of a hollow cylinder is given by

$$A = area\ of\ a\ rectangle = h \times 2\pi r = 2\pi rh$$

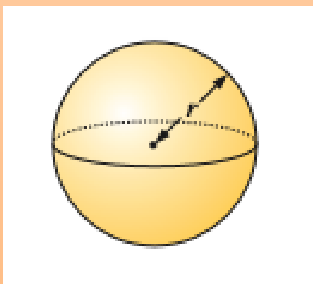
If the cylinder is open at one end like a cup then, $A = 2\pi rh + \pi r^2$

If the cylinder is solid, like a piece of chalk, then, $A = 2\pi rh + 2\pi r^2$

Spheres

The surface area of a sphere is given by

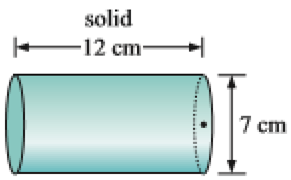
$$A = 4\pi r^2$$



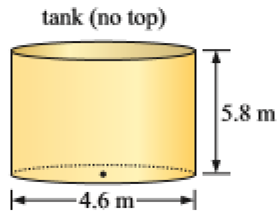
Exercise 8

1. Find the surface area of the following shapes:

a)



b)



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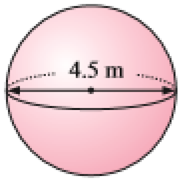
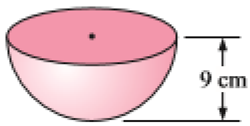


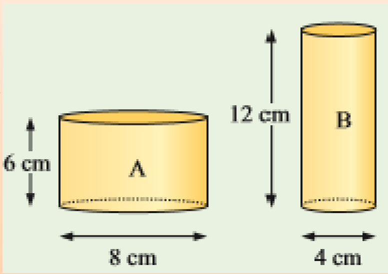
d)



2. Using the formula for the surface area of sphere, find the surface area of a soccer ball of diameter 20 cm.
3. An 8 cm by 10 cm rectangle has the same perimeter as an isosceles triangle with base 10 cm and equal sides 13 cm. Which figure has the greatest area and by how much?
4. Determine how much paint is required to paint the outside of a cylindrical tank 12 m long with a diameter 10 m if each litre of paint covers 15m^2.

Volume

Consider the cylindrical glasses A and B.
Deng says that both glasses occupy the same space, since glass A is twice as wide as glass B but only half as high.

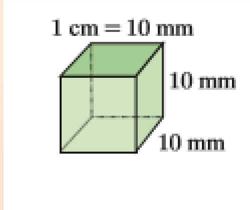


- Do you agree with Deng?
- If not, which glass do you think occupies more space?

These are the kind of questions we will be addressing in this section.

The volume of a three-dimensional object is the amount of space it occupies. This space is measured in cubic units.
We measure volume in **cubic units**.

For example, cubic centimetres (cm^3), cubic metres (m^3)
From the diagram alongside, we can see that



$1000\ mm^3 = 1\ cm^3$

$10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$ but $10\ mm = 1\ cm$. Therefore

$$1cm \times 1cm \times 1cm = 1cm^3 = 1000mm^3$$

Volume Conversions:

- Convert $23cm^3$ to mm^3

$$1\ cm^3 = 10^3\ mm^3 = 1000\ mm^3$$
$$23 \times 1000 = 23\ 000\ mm^3$$

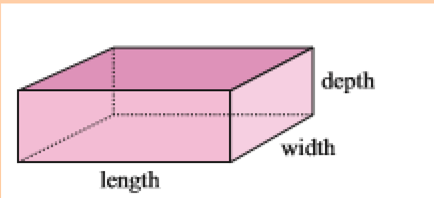
- Convert $24\ 000cm^3$ to m^3

$$1\ m^3 = 1\ 000\ 000\ cm^3 \text{ since } 1\ \text{m} = 100\ \text{cm}$$
$$24\ 000 \div 1\ 000\ 000 = 24 \times 10^3 \div 10^6 = 0.024\ m^3$$

Volume formulae

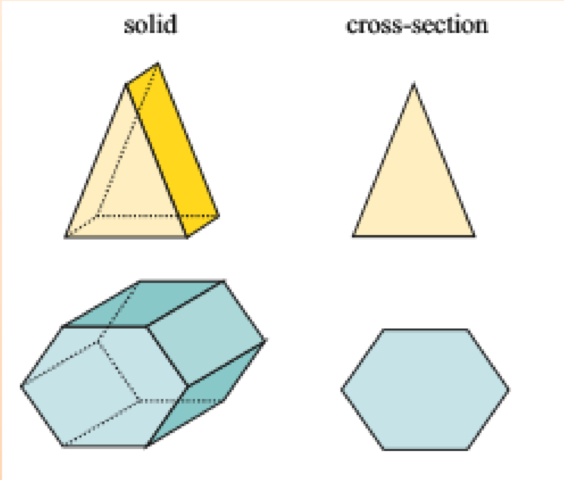
A. Rectangular prism

$$Volume = length \times width \times depth(height)$$



B. Solids of uniform cross section

Solids like the triangular prism or the hexagonal prism shown **have uniform cross section because any vertical slice made parallel to the front face will be the same shape and size of the front face.**



Therefore for any solid with a uniform cross section:

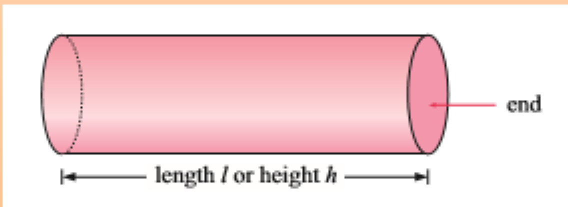
Volume = area of cross section × length

A cylinder has a uniform cross section, in this case a circle.

So its volume is given by:

Volume = area of circle × length

$= \pi r^2 \times l$ **or** $\pi r^2 \times h$



Therefore, volume of a cylinder is given by:

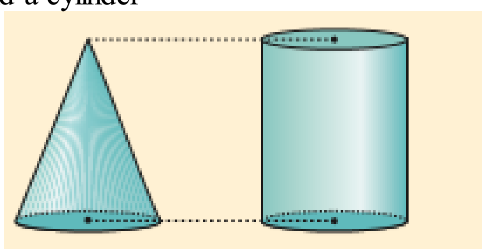
$$V = \pi r^2 l = \pi r^2 h$$

C. Pyramids and Cones

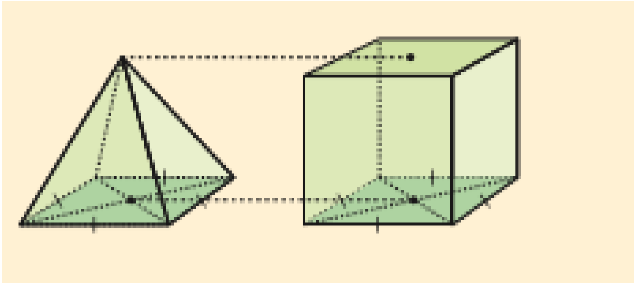
Task: To find the formula for the volume of pyramids and cones

We will compare the volumes of these solids with those of solids of uniform cross sections that have the same base and same height.

- A cone and a cylinder



- A square based pyramid and a square based prism (square based cuboid)



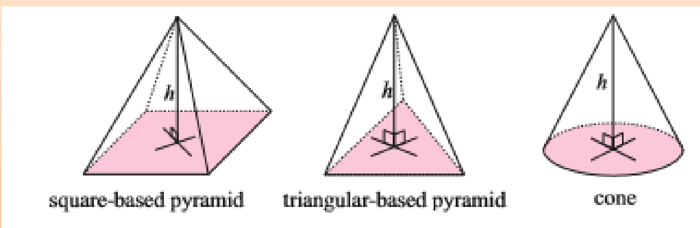
Steps:

1. Use printnets.com to get a printable sheet containing the nets for cone and cylinder of radius 4 cm and height 6 cm and a pyramid and square based cuboid of length 8 cm and height 6 cm.
2. Print the nets, cut them out and construct each of the solids using sticky tape to hold the models together.
3. Fill the model of the cone with sand/soil completely, then pour contents into the cylinder.
4. Repeat step 3 until cylinder is full. How many times do you need to fill up the cone in order to fill the cylinder?
5. Write an expression for the volume of the cone in terms of the volume of the cylinder.
6. Repeat steps 3-5 for the square based pyramid and cuboid.
7. Given that the volume of a solid with uniform cross section is

$$V = area\ of\ base \times height,$$

find a formula for the volume of a cone and a pyramid.

Pyramids and cones have a flat base and taper at a point called the **apex.** They do not have a uniform cross section.



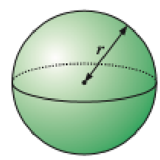
For such solids, their volume is given by the formula

$$V = \frac{1}{3}\textbf{(base area} \times \textbf{height)}$$

D. Spheres

Volume of a sphere is given by

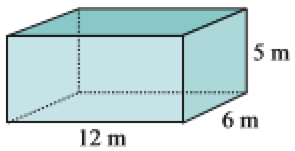
$$V = \frac{4}{3}\pi r^3$$



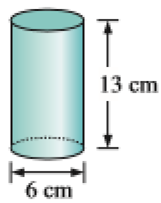
Exercise 9

Find the volume of the following:

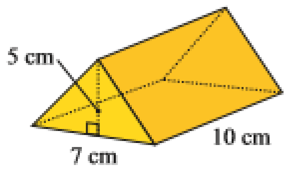
1. a)



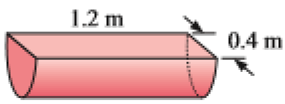
b)



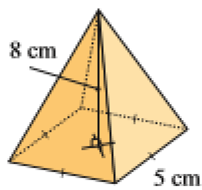
c)



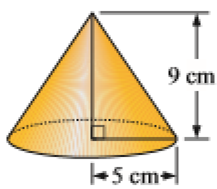
d)



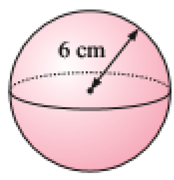
e)



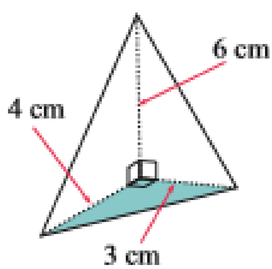
f)



g)

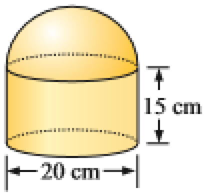


2. A triangular based pyramid is shown. Find its volume.



3. Find the volume of the figures below:

a)



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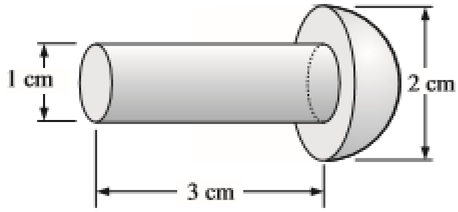


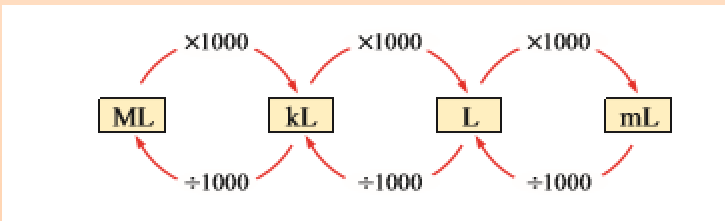
Capacity

The **capacity** of a container is the amount of material (solid or fluid) that it can contain.

The units of capacity are: mililitres(mL), litres(L) , kilolitres(KL) and megalitres(ML)

$$1\text{ L} = 1000\text{ mL}$$
$$1\text{ kL} = 1000\text{ L} = 1\,000\,000\text{ mL}$$
$$1\text{ ML} = 1000\text{ kL} = 1\,000\,000\text{ L}$$

Capacity conversions:



Example:

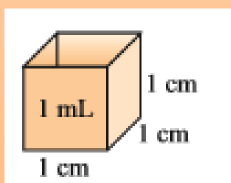
- Convert 4500mL to L

  $4500 \div 1000 = 4.5\text{ L}$

- Convert 6.5 kL to mL

  $6.5 \times 1000 \times 1000 = 65\,000\,000\, mL$

Volume and Capacity



There is a close relationship between volume and capacity

1 mL of fluid will fill a cube of 1 cm × 1 cm × 1 cm

1 cm^3 holds **1 mL**

In summary,

$$1\ mL = 1\ cm^3$$
$$1\ L = 1000\ cm^3$$
$$1\ kL = 1\ m^3$$

Example:

- Convert 4500 cm^3 to L

  $4500 \div 1000 = 4.5L$

- Find the capacity in kilolitres of a cylindrical container tank of height 3 m and diameter 4 m.

  Volume = base area x height

  $= \pi \times 2^2 \times 3$

  $= 12\pi = 37.70m^3$

  Therefore since $1m^3 = 1kL$, then capacity =**37.7kL**

Task: Volume and Capacity

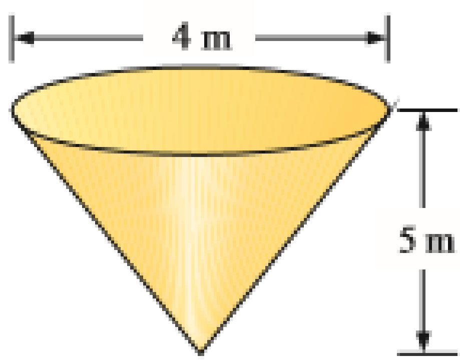
In pairs design a container that holds 1L, be as imaginative as you can.

Exercise 10

1. Convert

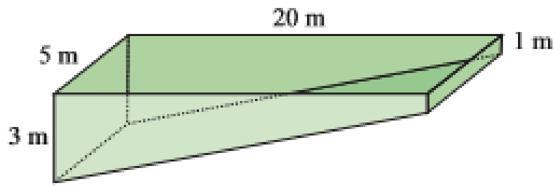
   a) 250 mL to cm^3 b) 18.5 m^3 to kL c) 4100 cm^3 to L

2. A conical tank has a diameter] 4 m and height 5 m. How many litres of water could it contain?



3. A bowl is hemispherical in shape with an internal diameter of 30 cm. How many litres of water can it contain?

4. Find the number of litres of water that would be required to fill the swimming pool with the dimensions shown below.



5. If a sphere has a surface area of 100 cm^2, find to 1 decimal place:
   a) its radius b) its volume

UNIT 3: GEOMETRY AND TRIGONOMETRY

1. Angles

If we look carefully, we can see **angles** in many objects and situations like the framework of buildings, the pitches of roof structures, the steepness of roads and the positions of boats from a harbor and airplanes from an airport.

The measurement of angles dates back more than 2500 years and is still very important today in architecture, building, surveying, engineering, navigation, space research and many more industries.

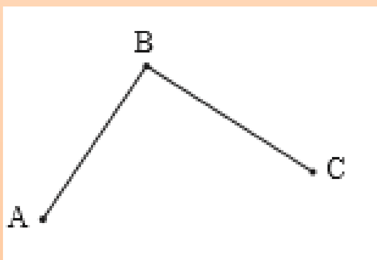
2. Points and lines

We use a **point** to mark a location or position. Examples of points are

- The corner of your desk
- The tip of a pair of compasses

Points do not have size. **In geometry they are represented by a small dot and labelled with a capital letter.**

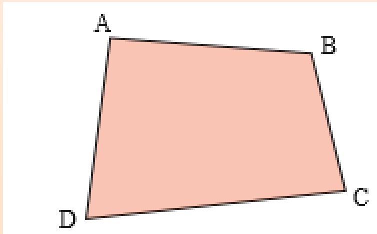
For example:



The letters A, B and C identify points.

3. Figures and vertices

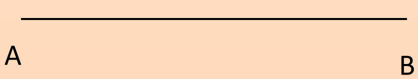
A figure is a drawing which shows points we are interested in. The figure below contains 4 points which have been labelled A, B, C and D.



B is a **vertex** of the figure ABCD and the **vertices** of the figure are A, B, C and D.

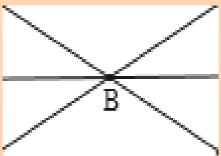
4. Straight lines

A **straight line** is a continuous infinite collection of points with no beginning or end which lie in a particular direction.



The diagram alongside shows the **line AB**

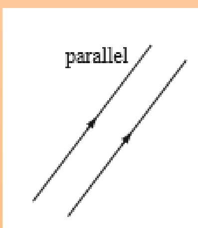
If three or more lines intersect at the same point, we say that the lines are **concurrent**.

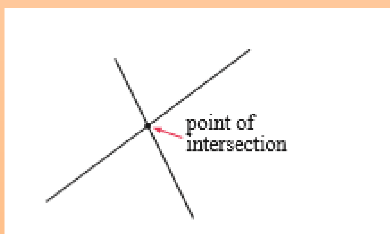


These lines are concurrent at B.

5. Parallel and intersecting lines

In mathematics, a plane is a flat surface like a table top of a sheet of paper. It goes on indefinitely in all directions. Two straight lines in the same plane may either be **parallel** or **intersecting**. Arrow heads are used to show parallel lines. **Parallel lines are lines a fixed distance apart and never meet.**

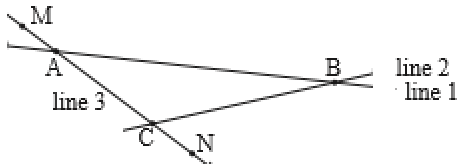




Exercise 1: In groups, discuss the following questions

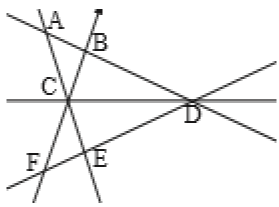
1. Give two examples in the classroom of:
   a) a point b) a line c) a flat surface

2. Name the point of intersection of:
   a) line 1 and line 2
   b) line 2 and line 3
   c) AB and MN



3. Draw a different diagram to fit each statement:
   a) C is a point on AB b) AB and CD meet at point X
   c) Point A does not lie on BC d) X, Y and Z are collinear

4. a) Name line AC in two other ways
   b) Name two different lines containing point B.
   c) What can be said about:
      i) points A, B and D
      ii) lines BF and AD
      iii) lines FE, CD and AB

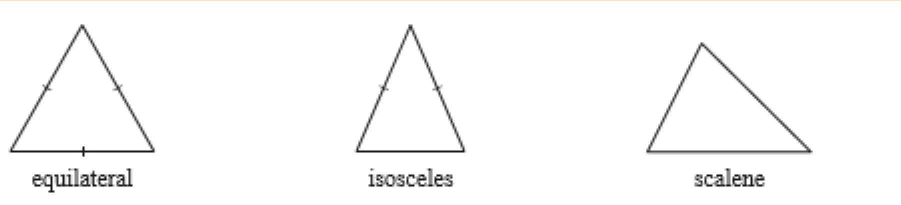


6. Triangles

Triangles may be classified according to the measure of their sides or the measure of their angles.

Classification by sides

A triangle is equilateral if all its sides are equal in length
A triangle is isosceles if at least two of its sides are equal in length
A triangle is scalene if none of its sides are equal in length.



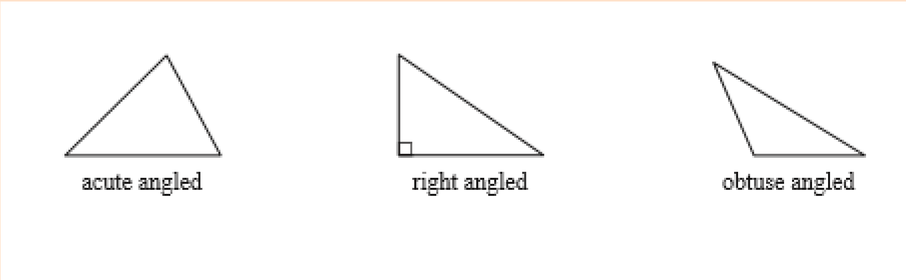
Classification by Angles

The size of the greatest angle dictates this classification.

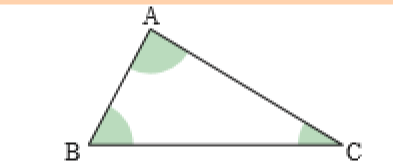
A triangle is acute angled if the greatest angle is acute.

A triangle is right angled if the greatest angle is a right angle (90^0)

A triangle is obtuse angled if the greatest angle is obtuse.

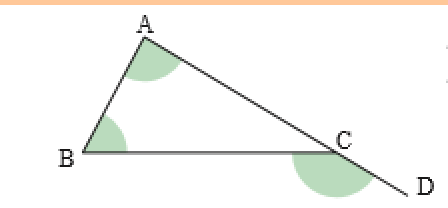


When we talk about the angles of a triangle we actually mean the **interior angles** or the angles *inside* the triangle.



The shaded angles are the interior angles of triangle ABC

If we extend a side of the triangle we create an **exterior** angle.



Angle BCD is an exterior angle of triangle ABC.

All triangles have 6 exterior angles.

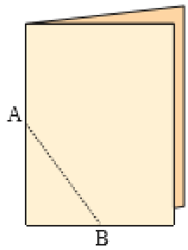
Isosceles Triangles:

Task: Making an Isosceles Triangle

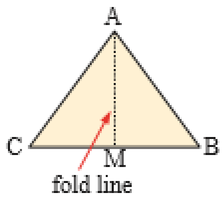
Working in pairs, get a clean sheet of paper.

Fold it down the middle. Draw a straight line AB as shown in the diagram below.

Then, with the 2 sheets pressed tightly together, cut along line AB through both sheets.



Keep the triangular piece of paper. When you unfold it, you should obtain the isosceles triangle ABC shown.



Discussion

In the triangle ABC above, explain why:

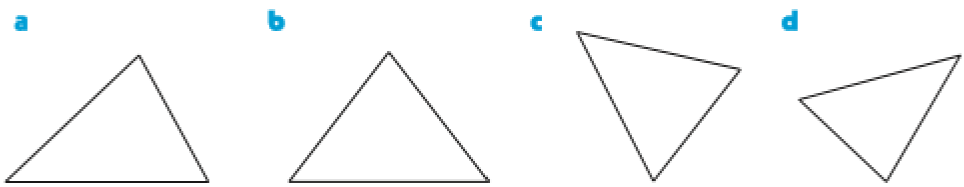
- Angle ABC=Angle ABC
- M is the midpoint of BC
- AM is at right angles to BC

From the task above, we can conclude that in any iscosceles triangle,

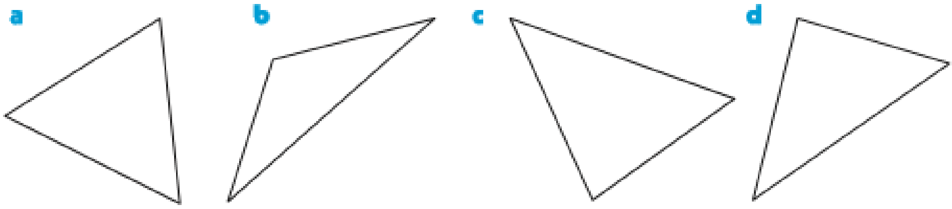
- The base angles are equal
- The line joining the apex to the midpoint of the base is perpendicular to the base.

Exercise 2: In pairs

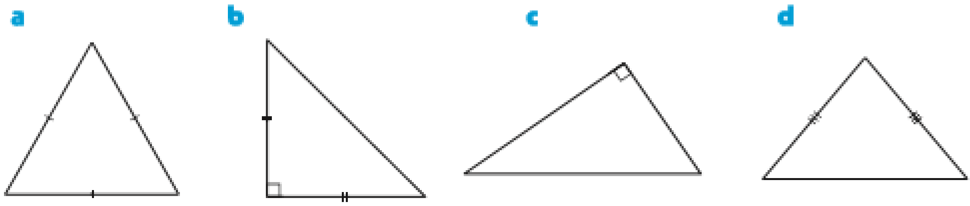
1. Measure the lengths of the sides of these triangles. Use your measurements to classify each as equilateral, isosceles or scalene.



2. Measure the sizes of the angles of these triangles. Use your measurements to classify each as acute, obtuse or right angled.



3. Classify the following triangles according to sides and angles. Each triangle should have two descriptions.



4. Explain why it is not possible to have a triangle which has
   a) two obtuse angles b) one obtuse angle and one right angle

Task: investigating the angles of a triangle:

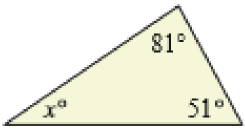
Work individually,

1. Construct a triangle ABC, clearly marking the measurement of each side
2. Measure and mark each of its interior angles, A^0, B^0 and C^0. Do they add up to 180^0?
3. Measure the exterior angle for angle C? Is it equal to the sum of A^0 and B^0?
4. Compare your results with other members of the class, what can you conclude?

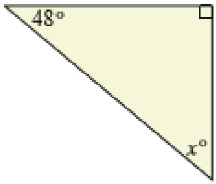
Exercise 3

1. Find x in the following, giving brief reasons:

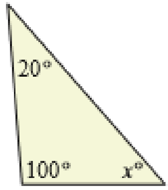
a)



b)

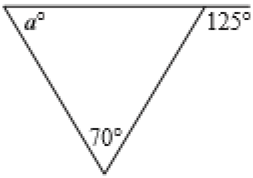


c)



2. Find a in the following, giving brief reasons:

a



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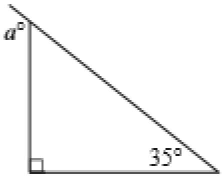
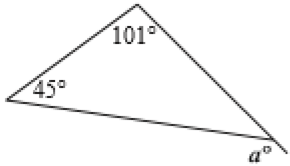


c



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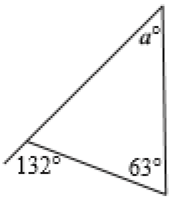
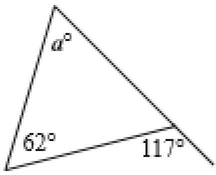
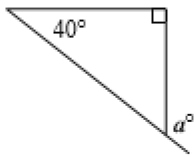


e

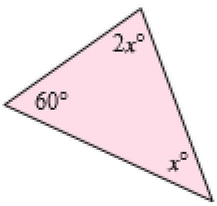


f

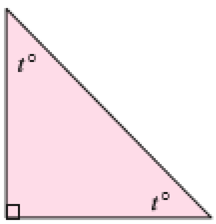


3. Find the unknowns in each triangle, giving brief reasons for your answers.

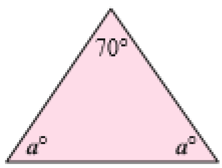
a



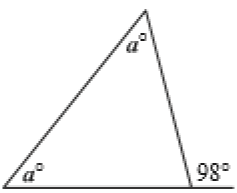
b



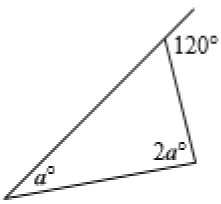
c



d



e



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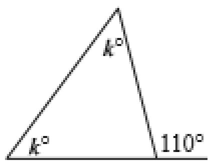
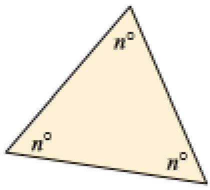
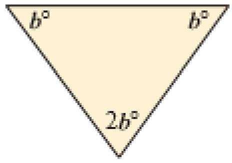


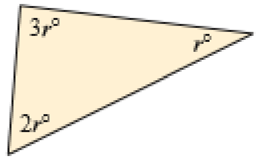
g



h

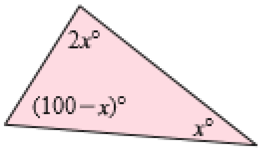


i



4. Find x given:

a



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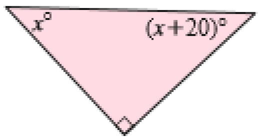
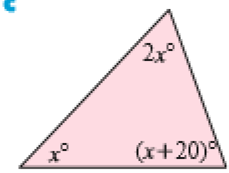


c



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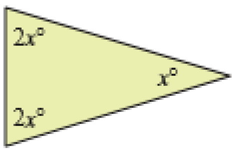

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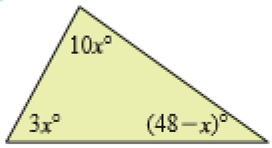
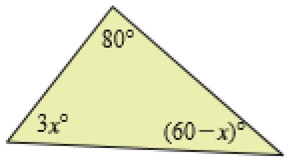
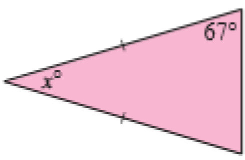


f

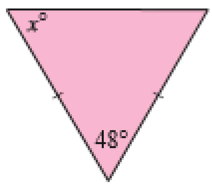


5. Find the unknown values in the following triangles

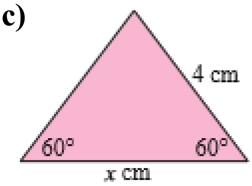
a)



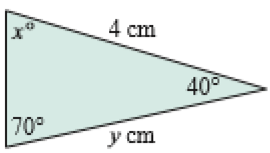
b)



c)



d)



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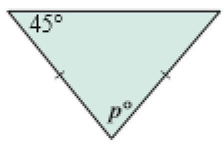

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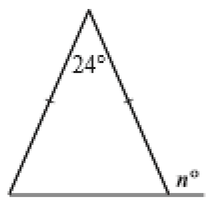
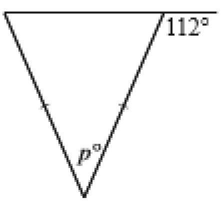
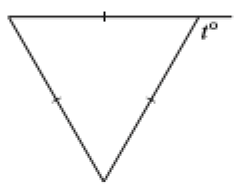


g)

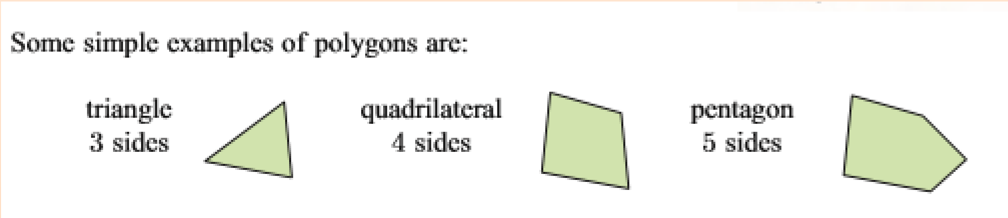


h)



Polygons

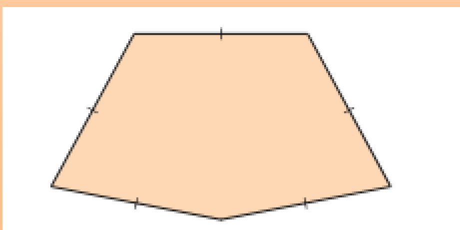
A shape that is drawn on a flat surface or plane is called a plane figure.
If the shape has no beginning or end it is said to be closed.
A polygon is a closed plane figure with straight edges that do not cross.



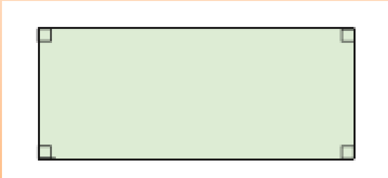
Polygons are named according to their number of sides. The table below shows the names of different polygons.

Number of Sides	*Polygon Name*
3	**Triangle**
4	**Quadrilateral**
5	**Pentagon**
6	**Hexagon**
7	**Heptagon**
8	**Octagon**
9	**Nonagon**
10	**Decagon**

A regular polygon has sides of **equal length** and **angles of equal measure.**
This polygon is not regular even though its sides are equal in length because its angles are not equal.



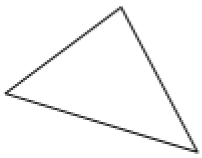
This polygon is not regular even though its angles are equal. Its sides are not equal in length.



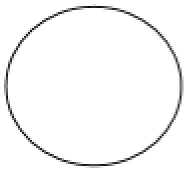
Discussion Questions

1. Which of these figures can be classified as a polygon?

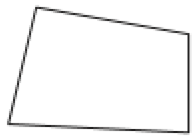
a)



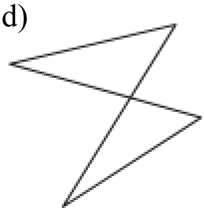
b)



c)



d)

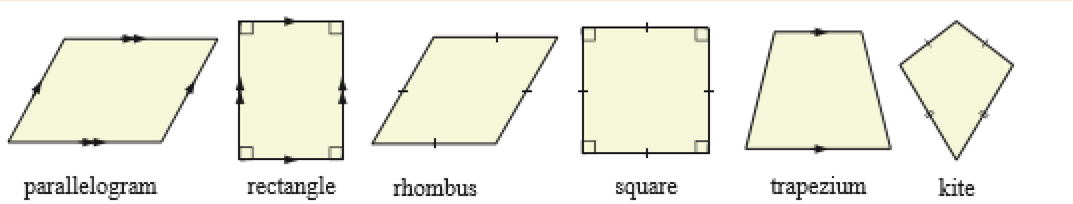


Quadrilaterals

A quadrilateral is a polygon with four sides.

There are six special quadrilaterals

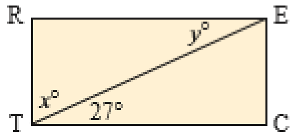
- A **parallelogram** is a quadrilateral which has opposite parallel sides.
- A **rectangle** is a parallelogram with four equal angles of 90°.
- A **rhombus** is a quadrilateral in which all sides are equal.
- A **square** is a rectangle with four equal angles of 90°.
- A **trapezium** is a quadrilateral which has a pair of parallel sides.
- A **kite** is a quadrilateral which has two pairs of adjacent equal sides.



Exercise 5 (To be done in groups)

1. Solve the following problems:

a) RECT is a rectangle. Find the values of x and y

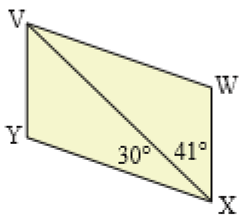


b) PARM is a parallelogram. Find the size of:

i. Angle PMR
ii. Angle ARM
iii. Angle PAR

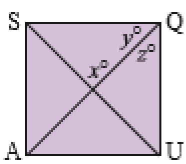
c) VWXY is a parallelogram. Find the size of:

i. angle WVX
ii. Angle YVX
iii. VYX
iv. VWX



d) SQUA is a square. Find the values of:

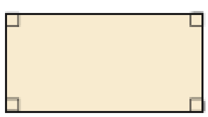
i. x ii. y iii. z



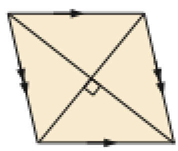
2. Using information given in the diagram, name the following quadrilaterals. Give brief reasons for your answer.

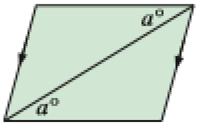
a)

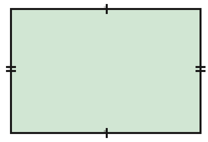
b)

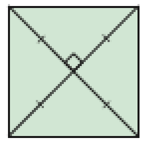


c)



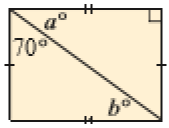
d) 

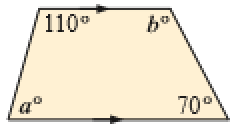
e) 

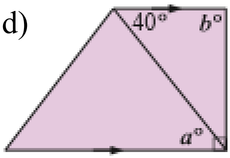
f) 

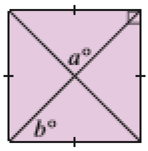
3. Use the information given to name the quadrilateral and find the values of the variables:

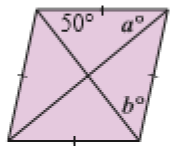
a) 

b) 

c) 

d) 

e) 

f) 

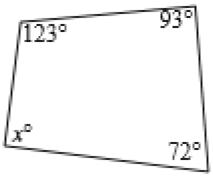
Task: Angles of a Quadrilateral

- Draw any quadrilateral on a piece of paper.
- Divide the quadrilateral into triangles. What is the minimum number of triangles you can divide the quadrilateral into?
- Using the triangles, what is the sum of the angles in the quadrilateral?
- Now try for other polygons
- Generalise your result

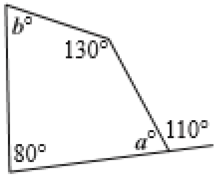
Exercise 6

1 Find the values of the variables, giving brief reasons for your answers:

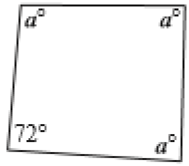
a



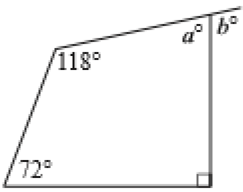
b



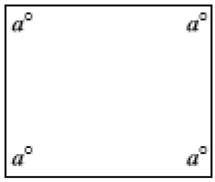
c



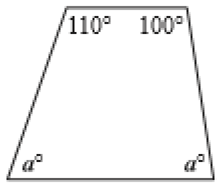
d



e



f



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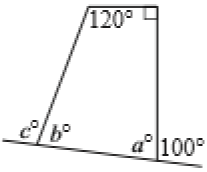
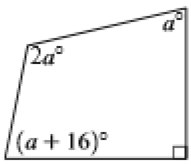
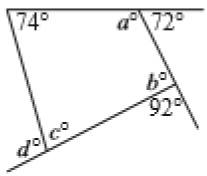


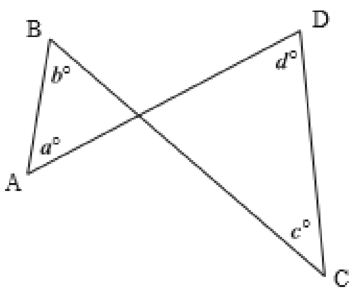
h



i



2



Consider the illustrated quadrilateral ABCD.

a Why is it not a polygon?

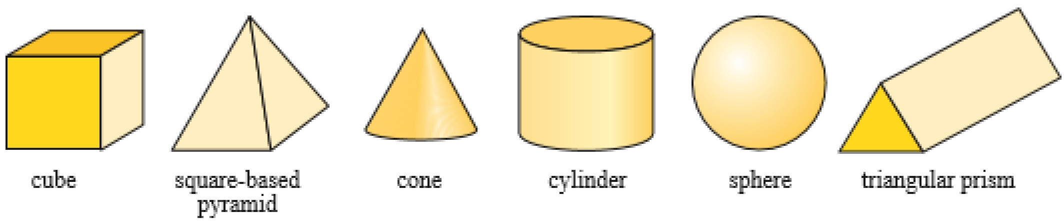
b Explain why $a + b = c + d$.

c Show that $a + b + c + d$ must always be less than 360^o.

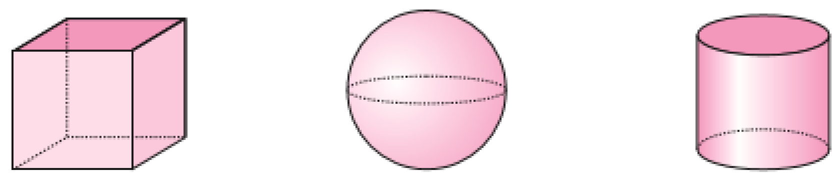
Solids

A **solid** is a three dimensional body which occupies space.

The diagrams below show a collection of solids. Each solid has three dimensions: *length, width* and *height.*



The boundaries of solids are called **surfaces**. These may be flat surfaces, curved surfaces or a mixture of both.



A cube is bounded by six flat surfaces.

A sphere is bounded by one curved surface.

A cylinder is bounded by two flat surfaces and one curved surface.

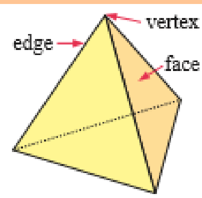
When we draw solids, we use dashed lines to show edges which are hidden at the back of the solid.

Polyhedra

A **polyhedron** is a solid which is bounded by flat polygonal surfaces only. The plural of polyhedron is **polyhedra.**

Examples of polyhedral are cubes and pyramids. Spheres and cylinders are **not** polyhedra.

- Each flat surface of a polyhedron is called a **face** and has the shape of a polygon.
- The point at which edges meet is called a **vertex**. The plural of vertex is **vertices**



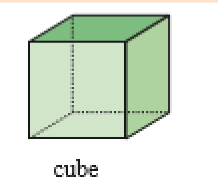
The solid shown is a triangular based pyramid

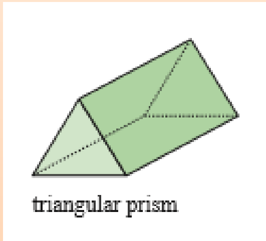
Or **tetrahedron**. It has 4 vertices, 4 faces and 6 edges.

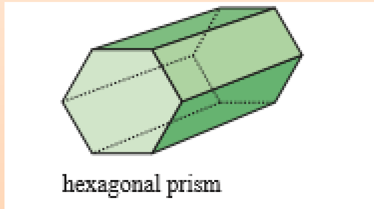
Prisms

A **prism** is a solid with a uniform cross section that is a polygon.

Examples of prisms:



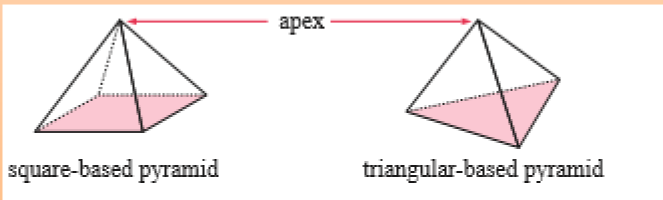




Pyramid

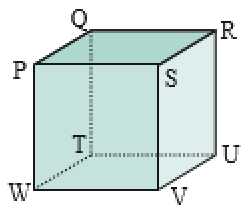
A pyramid is a solid with a polygon for a base and triangular faces which come from the edges of the base to meet at a point called the **apex.**

Examples of pyramids:



Exercise 7: Working in pairs answer the following questions

1. a) Name all the vertices of this cube.
b) Name all the faces of this cube.
c) Name all the edges of this cube.
d) Name all the edges parallel to SV.
e) Name all the edges that concurrent at P.
f) Name a face which is parallel to face PQRS.



2. Draw a solid which has:
a) only a curved surface.
b) a curved and a flat surface.
c) two flat and one curved surface.

Transformation geometry

A change in the size, shape, orientation or position of an object is called a **transformation**.

Reflections, rotations, translations and enlargements are all examples of transformations. We can describe these transformations mathematically using **transformation geometry**.

The original figure is called the **object** and the new figure is called the **image.**

The transformations to be considered in this section are:

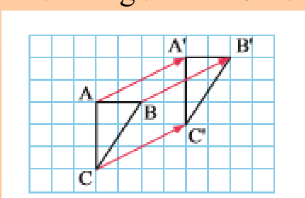
- Translations, where every point on an object moves a fixed distance.
- Rotations, where we turn objects about a point.
- Reflections, where we reflect objects along a mirror line.
- Enlargements, where objects are made bigger or smaller relative to centre of enlargement.

Translation

A translation is a transformation in which every point of a figure moves a fixed distance in a given direction. The distance the figure moves is given in terms of the steps horizontally and vertically.

Example:

Triangle ABC is translated to a new position by sliding every point of ABC the same distance in the same direction. We call the new triangle A'B'C'.
Triangle ABC is the object and triangle A'B'C' is the image.



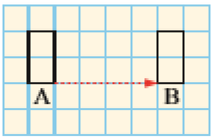
We give the details of a translation by using a **translation vector.**
For instance, each point of triangle ABC has moved 4 units to the right and 2 units upwards.

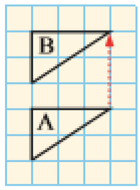
The translation vector is therefore $\begin{pmatrix}4\\2\end{pmatrix}$

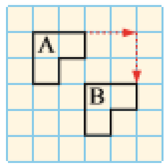
Note that the translation vector that can move triangle A'B'C' back to triangle ABC is $\begin{pmatrix}-4\\-2\end{pmatrix}$.

Exercise 8

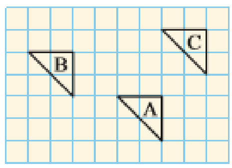
1. The object A has been translated to give an image B in each diagram. Specify the translation in each case using a translation vector.

   a)


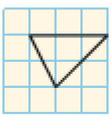
   b)


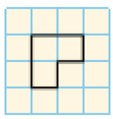
   c)


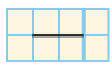
2. The figure A has been translated to B, then B has been translated to C.
   a) Give the translation vector from A to B.
   b) Give the translation vector from B to C
   c) What translation vector would move A to C?



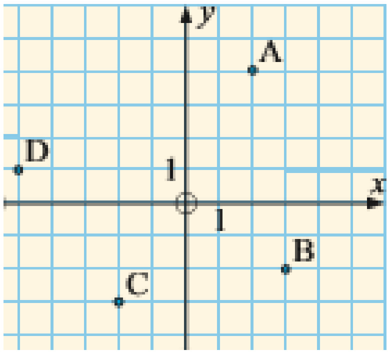
3. Copy the figures below onto squared paper and translate using the vector given:

   a.  $\begin{pmatrix}2\\3\end{pmatrix}$

   b)  $\begin{pmatrix}3\\-2\end{pmatrix}$

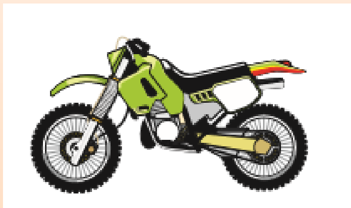
   c)  $\begin{pmatrix}3\\-2\end{pmatrix}$

4. a) Write down the coordinates of A, B, C and D.
   b) Each point is translated 5 units to the right and 2 units up. What are the coordinates of the image points A', B', C' and D'?

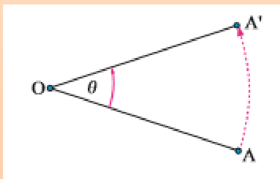


5. Draw triangle ABC where A is (-1,3), B is (4, 1) and C is (0, -2).
   a) Translate the figure with translation $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$
   b) State the coordinates of the image vertices A', B' and C'.

Rotation



When a wheel moves about its axle, we say that the wheel *rotates*.
The centre point on the axle is called the **Centre of Rotation.**
The angle through which the wheel turns is called the **Angle of rotation.**



Rotations are transformations in which one point called O is fixed and all other points are rotated through the Same angle θ about O.
O is called the **Centre of the rotation** and θ is known as the **Angle of rotation.**

Notice that OA=OA' and AA' is an arc of a circle with Centre O.
To completely describe a rotation we need to know:

- the centre of rotation.
- the direction of the rotation (clockwise or anticlockwise).
- the angle of rotation.

Task: Rotations using tracing paper.

a) Draw triangle ABC with A(1, 1) B(1, 3) and C (3, 3). Let the origin (0, 0) be the centre of rotation. Rotate triangle ABC 90^0 clockwise

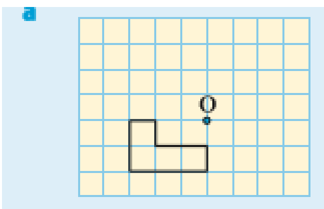
Steps:

- On tracing paper, draw triangle ABC and mark the centre of rotation O. Draw a line segment from O to point A.
- Measure 90^0 in a clockwise direction from O and draw a line segment of the same length as OA. Label this as OA^1.
- Turn your tracing paper so that OA fits onto OA^1. You can now see the position of the image
- Trace the image from the tracing paper to your book and label the image
- Write down the coordinates of the image.

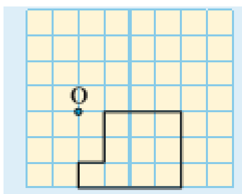
2. Follow steps in question 1 above to find the image of triangle ABC after a rotation of 90^0 anti-clockwise about the point (-1, 1)

3. Rotate the given figures about O through the angle indicated:

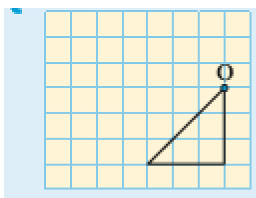
A.



180^0 clockwise



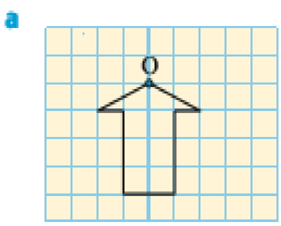
90^0 anticlockwise



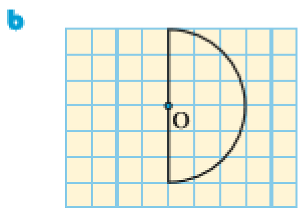
90^0 clockwise

Exercise 9

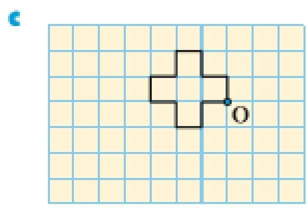
1. Rotate the figures through the angle given about O:



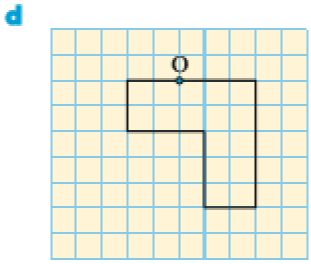
180^o

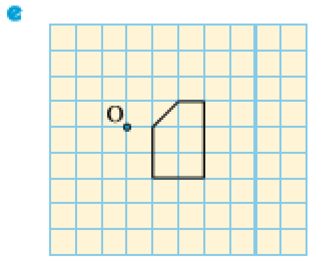


180^o

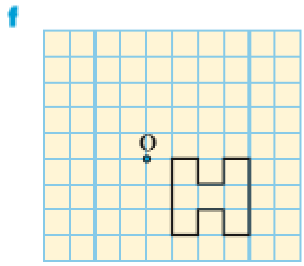


90^o anticlockwise



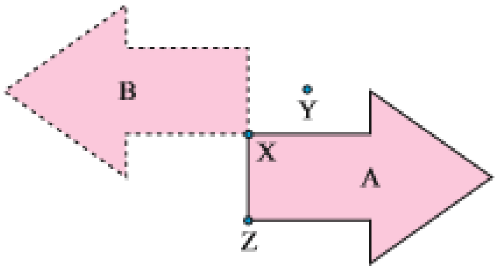


90° clockwise



90° anticlockwise

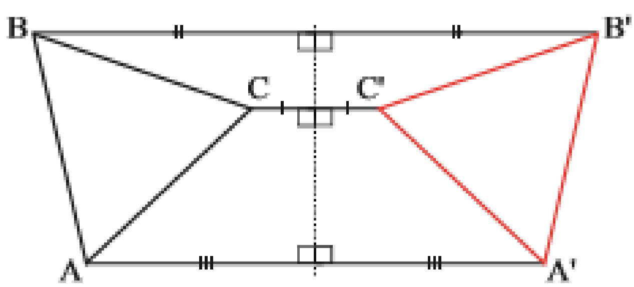
2. Figure B is the image of A after a rotation.
   a) Through how many degrees has A been rotated?
   b) Which point was the centre of rotation?



Reflection

We encounter reflections every day as we look in a bathroom mirror or look down at a stream or pond of water.

Consider the figure below:



mirror line

- The image and object are the same distance from the mirror line but on opposite sides.
- The line joining any image point to the corresponding point on the object is at right angles to the mirror line.
- All lengths and angles are the same size in the image as they were in the object.
- Points on the mirror line do not move.

The mirror line is at the perpendicular bisector of every point on the object and its corresponding point on the image

We can use these facts to help us draw reflections. It is easy to work on grid paper because we can count squares and see right angles.

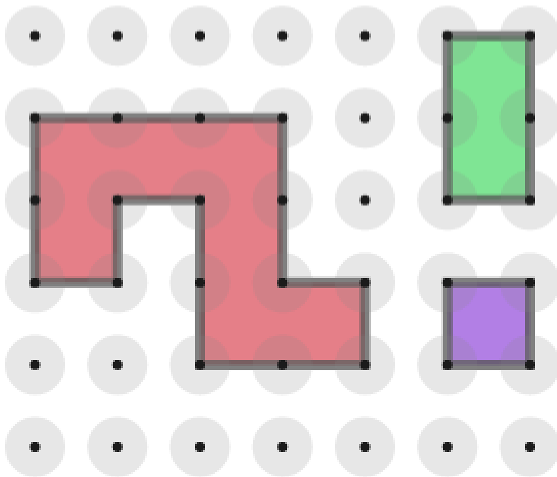
Task

1. Draw triangle ABC on squared paper or graph paper. A (2, 2), B (6, 2) and C (6, 5).
    a) Draw triangle A′B′C′ on the same axes, A′(-2, 2), B′(-6, 2) and C′(-6, 5)
    b) What do you notice?
    c) Reflect triangle ABC using the x-axis as the mirror line. Write the coordinates of the image.

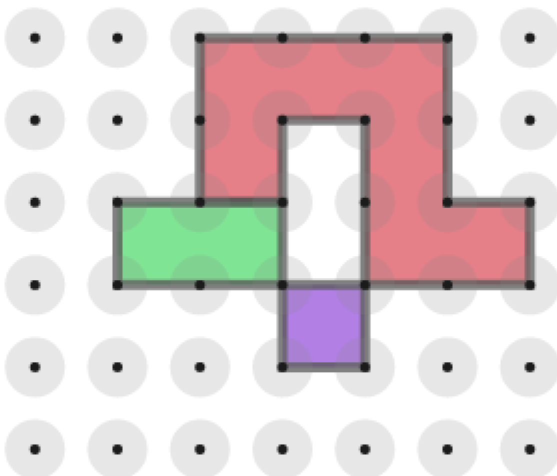
2. Draw rectangle PQRS, P (-2, -2), Q (-2, 2) R(2, 2) and S(2, -2).
    a) Reflect this rectangle using the y-axis as the mirror line. Write down the coordinates of the image.
    b) Reflect PQRS using the x-axis as the mirror line. Write down the coordinates of the image.

Task: Reflection Game

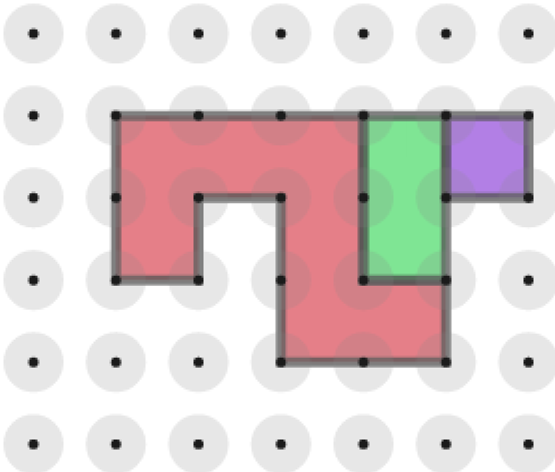
The three pieces below can be fitted together to make shapes with at least one line of symmetry.



The vertices of each piece must lie on grid points, and you must not overlap two pieces.



The pieces must be placed edge to edge, so this is not allowed.



This arrangement does not satisfy the criteria because the shape does not have a line of symmetry.

1. Can you find all the possible solutions? (There are more than six.)
2. How can you be sure you've found them all?

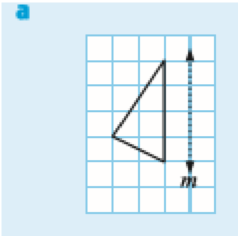
Here are some further questions to explore:

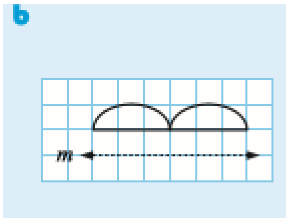
Design your own set of three shapes, with a total area of 10 square units, as above.

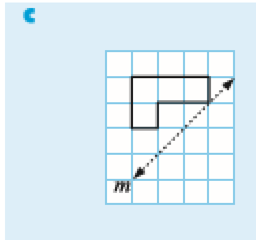
1. How many ways can they be arranged to make symmetrical shapes?
2. Can you find a set of three such shapes which can be arranged into more symmetrical shapes than those in the original problem?
3. Can you find three such shapes which can **never** be arranged to make a symmetrical shape?

Exercise 10

1. Reflect the following figures in the given mirror lines

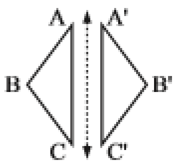




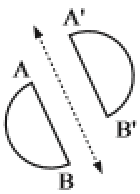


Which of the following transformations represent reflections?

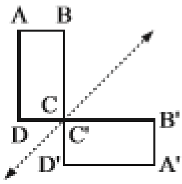
a



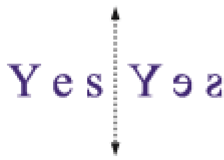
b



c



d



3. a) By plotting points on grid paper, complete the table alongside showing the images of the given points under a reflection in the x-axis

b) Use your table from (a) to complete the following

Statement: under a reflection in the x-axis, (a, b) maps

Onto (____________________

	P	P'
i	(5, 2)	
ii	(−3, 4)	
iii	(−2, −5)	
iv	(3, −7)	
v	(4, 0)	
vi	(0, 3)	

Enlargement

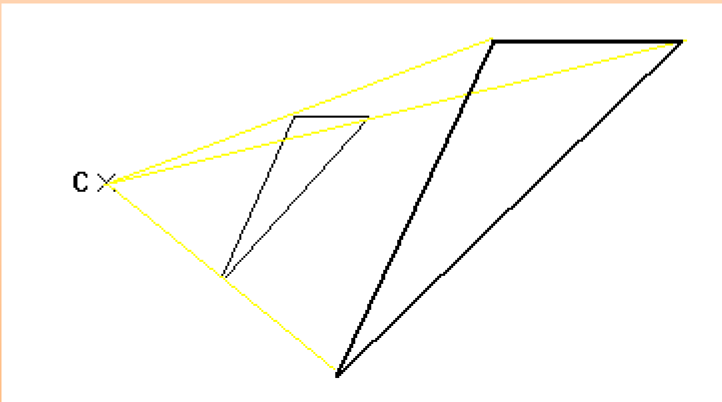
When you **enlarge** an object, you change its size but keep its original shape the same. In order to carry out enlargements, you need to know the scale factor and the centre of enlargement.

The **scale factor** tells us how much the object has been enlarged and can be positive, negative or fractional.

The **centre of enlargement** tells us the specific point from which the enlargement is being measured.

Example

The centre of enlargement is marked. Enlarge the triangle by a scale factor of 2.



***Investigation:* Enlargement**

Students should work in groups

1. Draw triangle ABC with A (2,1), B(4, 2) and C(3, 3)

a) Draw triangle $A'B'C'$ with $A'(4, 2)$ $B'(8, 4)$ and $C'(6, 6)$

b) Join A to A', B to B' and C to C' and mark the point where the lines meet. This point is known as the **centre of enlargement**.

c) What is the **scale factor of enlargement?**

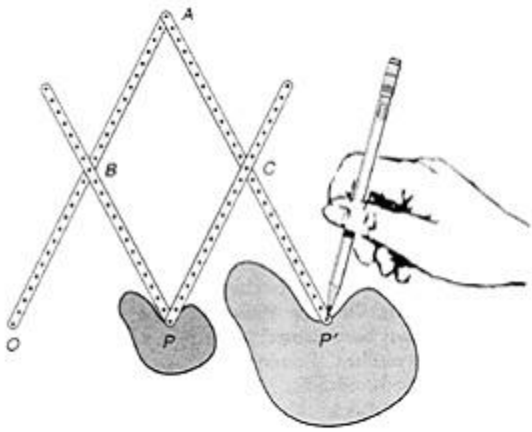
d) Find the coordinates of the image of triangle ABC after an enlargement with a scale factor of $\frac{1}{2}$ and the same centre of enlargement as above.

2. Draw rectangle WXYZ with W (1, 2), X (1, 3), Y (3, 3) and Z (3, 2)

a) On the same axes, draw W′X′Y′Z′ with W′ (-2, 4), X′ (-2, 6), Y′ (-6, -6) and Z' (-6, -4)

b) Join the object to the image as done in question 1(c) above. What are the coordinates of the centre of enlargement?
c) What is the scale factor?
d) Find the coordinates of rectangle WXYZ after an enlargement with scale factor -1 and the same centre of enlargement.

3. A **pantograph** is a mechanical device for enlarging and reducing figures by producing a similarity mapping. It can be constructed by computer software programs or constructed from four strips of wood, metal, or poster board that are drilled with equally spaced holes, as shown in the following figure. The arms of the pantograph are hinged at points A, B, C, and P so that they move freely. Point O is the projection point and should be held fixed. As point P traces the original figure, a pencil at point P' (its image) traces the enlargement. In this figure the arms are set so that point P' is twice as far from the projection point O as is point P, so the scale factor for this mapping is 2. To reduce a figure, the pencil is positioned at P and P' is moved around the boundary of the original figure. With the arms set as shown in the above figure, the scale factor for such a reduction is $\frac{1}{2}$. The pantograph can be changed to obtain different scale factors by adjusting the locations of points B and C. Build a pantograph and experiment with different settings of the arms.



Starting Points for Investigation

1. For an enlargement with a scale factor of 2, point B is half way between O and A, and point C is half way between A and P'. Where should points B and C be placed for an enlargement with a scale factor of 4? (Hint: To check your answer, use the pantograph to enlarge a unit square. The enlargement should have an area of 16 square units. Why?)
2. What happens to the scale factor for an enlargement as point B is moved toward O and point C is moved toward A?

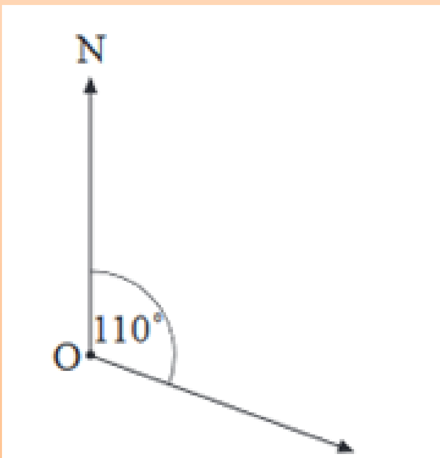
3. Where should points B and C be placed for an enlargement with a scale factor of 8?
4. What happens to the scale factor for an enlargement as point B is moved toward A and point C is moved toward P'?

Bearings

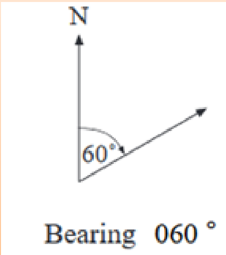
A bearing is a description of a direction. It is the number of degrees measured in a clockwise direction from north as seen from above. Convention, probably born out of the need to be quite clear when saying a bearing over a crackly aircraft radio or storm at sea, three figures are given for each bearing. So 90 degrees would be expressed as 090 degrees.

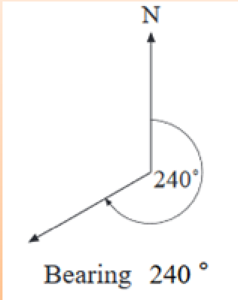
The four main directions are north (0°), east (090°), south (180°) and west (270°). The directions in between are north-east (045°), south-east (135°), south-west (225°) and north-west (315°).

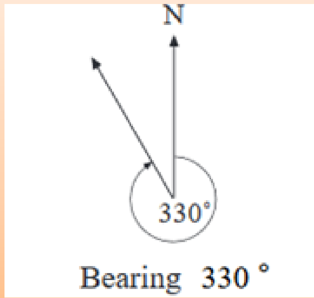
Bearings are a measure of direction, with north taken as a reference. If you are travelling north, your bearing is 000^0.
If you walk from O in the direction shown in the diagram, you are walking a bearing of 110^0.



Bearings are always measured clockwise from north, and are given as three figures, for example:

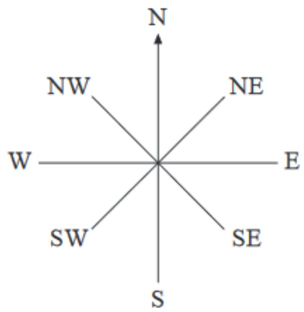






Task

In pairs, copy and complete the table.



Compass points	Bearing
N	
NE	
E	
SE	
S	
SW	
W	
NW	

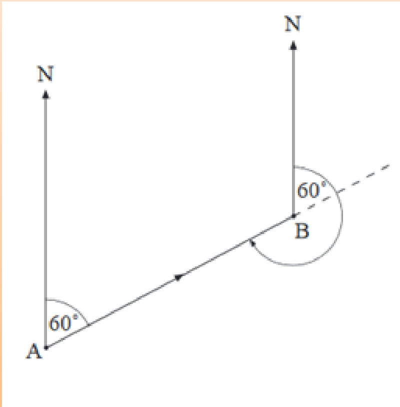
Example

A ship sails from A to B on a bearing of 060°. On what bearing must it sail if it is to return from B to A?

Solution:

The diagram shows the journey from A to B.

Extending the line of the journey allows an angle of 60^0 to be marked at B.
Bearing of A from B = $60^0+180^0=240^0$



Exercise 11: Work in pairs.

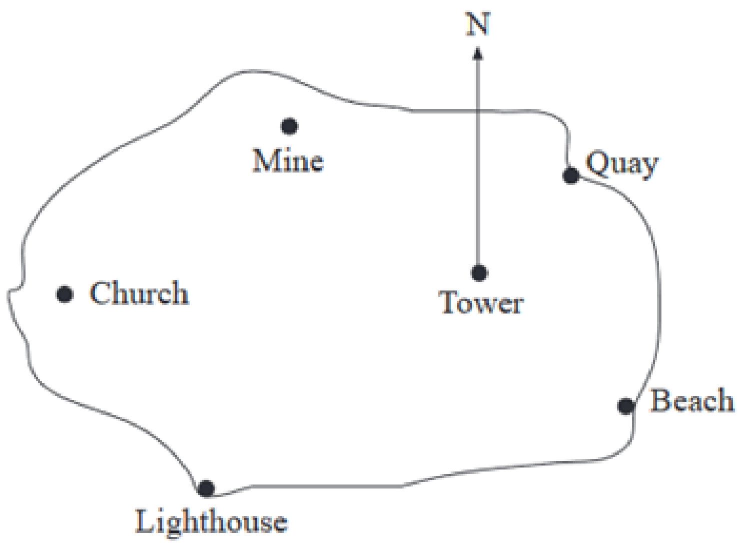
1. What is the three figure nearing of north-west?
2. What is the three figure bearing of south-east?
3. What is the three-figure bearing of north-east?
4. Achan is driving south-west. In his rear-view mirror he can see a tower directly behind him. What is the three figure bearing of the tower from his current position?
5. Keji is hiking in a north-westerly direction then he turns right (90°). What is the three figure bearing of her direction now?
6. Deng is swimming in a river. After swimming in a north westerly direction he turns around (180°). What is the three figure bearing of his direction now?
7. Garang is facing north-east. How many degrees does he have to turn through clockwise to face south?
8. Susan is facing south-east. How many degrees does she have to turn anticlockwise to face north-west?
9. Noi is facing north-west. How many degrees does he have to turn through clockwise until he is facing south?

Exercise 12

1. What angle do you turn through if you turn clockwise from?
   a) N to S

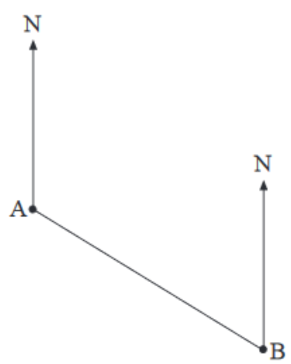
b) E to W
c) N to NE
d) N to SW
e) W to NW

2. The map of an island is shown below:



What is the bearing of the tower, from each place shown on the map?

4. The diagram shows the positions of two ships, A and B.
a) What is the bearing of ship A from ship B?
b) What is the bearing of ship B from ship A?



5. A ship sails NW from a port to take supplies to an oil rig. On what bearing must it sail to return from the oil rig to the port?

Scale drawings

Using bearings, scale drawings can be constructed to solve problems.

Example 1:

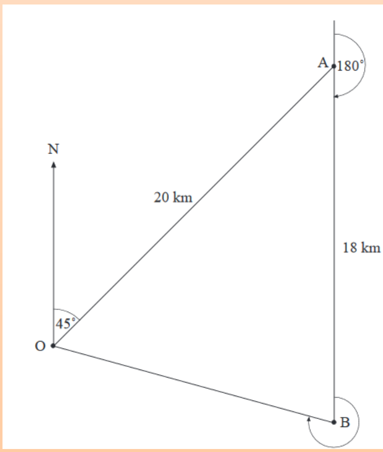
A ship sails 20 km NE, then 18 km S, and then stops.

a) How far is it from its starting point when it stops?
b) On what bearing must it sail to return to its starting point?

Solution:

The path of the ship can be drawn using a scale of 1 cm for every 2 km, as shown in the diagram.

Scale: 1 cm=2 km



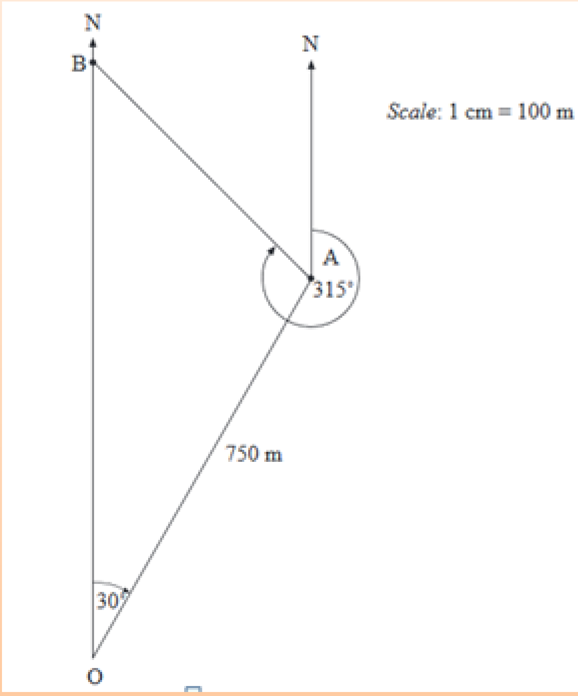
Example 2:

A man walks 750 m on a bearing of 030^0. He then walks on a bearing of 315^0 until he is due north of his starting point, and stops.

a) How far does he walk on the bearing of 315^0?
b) How far is he from his starting point when he stops?

Solution:

Using a scale of 1 cm to 100 m, a scale drawing can be drawn.



a) The distance AB represents the distance walked on a bearing of 315^0. It measures 5.4 cm which represents an actual distance of 530 m.

b) The distance OB represents the distance he is from his starting point. It measures 10.2 cm, representing an actual distance of 1020 m.

Exercise 13

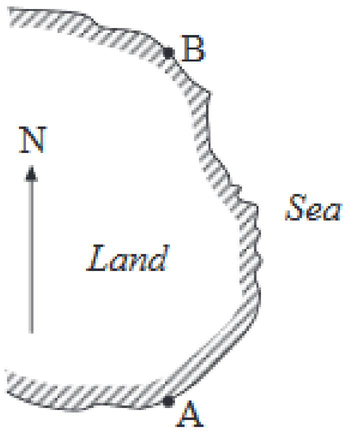
Discussing in your groups, individually draw the following scale diagrams in your books and answer the questions that follow.

1. A girl walks 80 m north and then 200 m east.
    a) How far is she from her starting position?
    b) On what bearing should she walk to get back to her starting position?

2. Andrew walks 300 m NW and then walks 500 m south and then stops.
    a) How far is he from his starting position when he stops?
    b) b) On what bearing could he have walked to go directly from his starting position to where he stopped?

3. An aeroplane flies 400 km on a bearing of 055^0. It then flies on a bearing of 300^0, until it is due north of its starting position. How far is the aeroplane from its starting position?

4. A captain wants to sail his ship from port A to port B, but the journey cannot be made directly. Port B is 50 km north of A. The ship sails 20 km on a bearing of 075^0, then sails 20 km on a bearing of 335^0 and drops anchor.
   a) How far is the ship from port B when it drops anchor?
   b) On what bearing should the captain sail the ship to arrive at port B?



Trigonometry

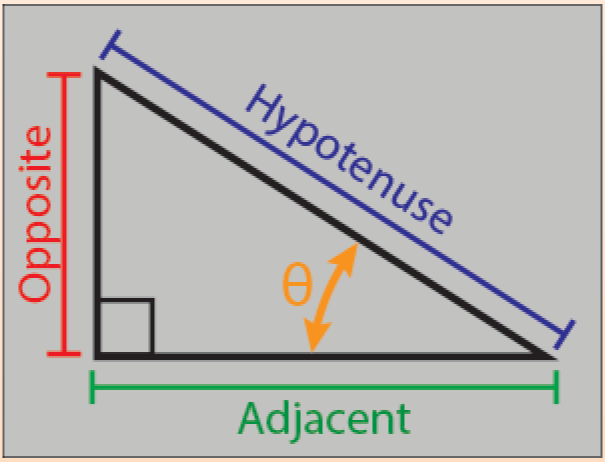
Trigonometry, as the name might suggest, is all about triangles.

More specifically, trigonometry is about right-angled triangles, where one of the internal angles is 90°. Trigonometry is a system that helps us to work out missing sides or angles in a triangle.

Right-Angled Triangles: A Reminder

A right-angled triangle has a single right angle. By definition, that means that all sides cannot be the same length. A typical right-angled triangle is shown below.

Important terms for Right-Angled Triangles



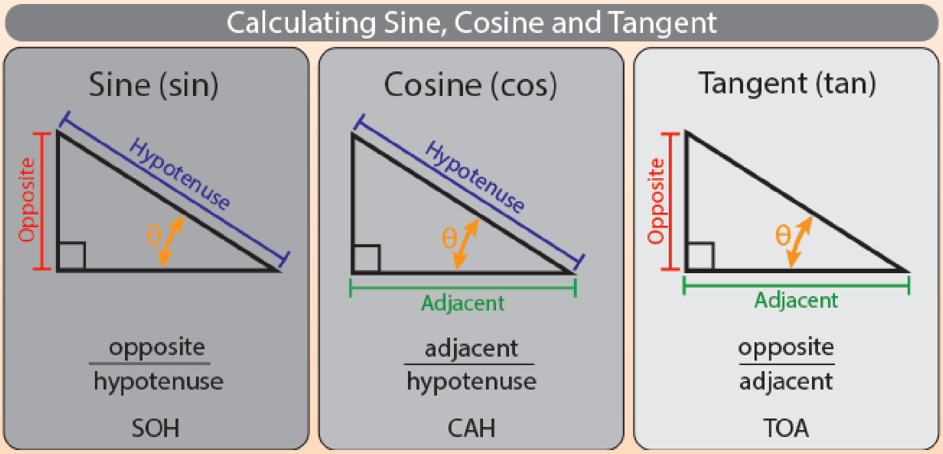
- The **right angle** is indicated by the little box in the corner.
- The other angle that we (usually) know is indicated by **θ**.
- The side opposite the right angle, which is the longest side, is called the **hypotenuse**.
- The side opposite θ is called the **opposite**.
- The side next to θ which is not the hypotenuse is called the **adjacent**.

Introducing Sine, Cosine and Tangent

There are three basic functions in trigonometry, each of which is one side of a right-angled triangle divided by another.

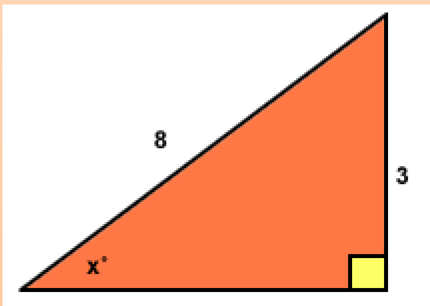
The three functions are:

Name	Abbreviation	Relationship to sides of the triangle
Sine	**Sin**	**Sin (θ) = Opposite/hypotenuse**
Cosine	**Cos**	**Cos (θ) = Adjacent/hypotenuse**
Tangent	**Tan**	**Tan (θ) = Opposite/adjacent**



Example

Calculate $x°$. Give your answer to one decimal place.



Solution

$\sin x° = \frac{3}{8}$

$\sin x° = 0.375$

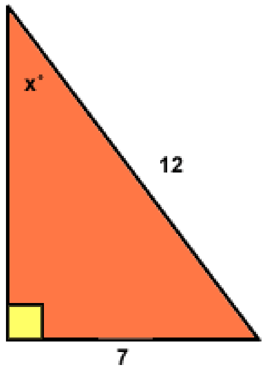
$\sin x° = 22.0°$ (to one decimal place)

Exercise 14

Use the same method in the following questions, but take care:
Sometimes you will use $\sin x°$, sometimes $\cos x°$ and sometimes $\tan x°$.
Hint: Show all working, especially when you use a calculator.

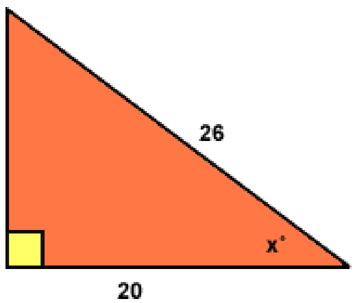
Calculate x°. Give your answer to one decimal place.

1



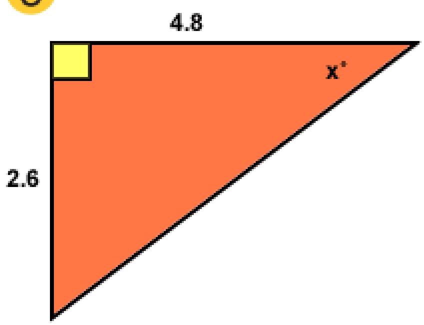
Calculate x°. Give your answer to one decimal place.

2

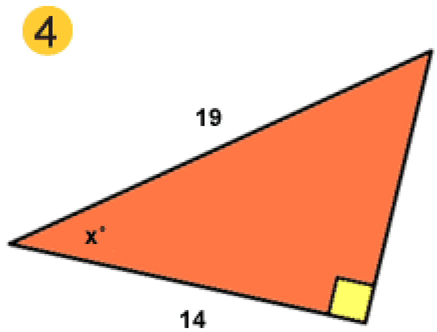


Calculate x°. Give your answer to one decimal place.

3



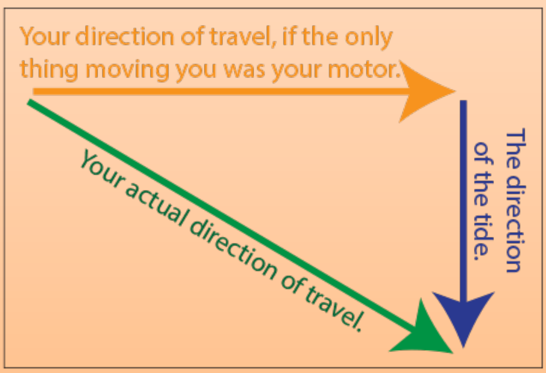
Calculate x°. Give your answer to one decimal place.



Trigonometry and Navigation

When you are sailing or cruising at sea, where you end up is affected by:

- The direction in which you steer;
- The speed at which you travel in that direction (i.e. the motor or wind speed); and
- The direction and speed of the tide.
- You can be motoring in one direction, but the tide could be coming from one side, and push you to the other. You will (ta-da!) need trigonometry to work out how far you will travel and in what precise direction.

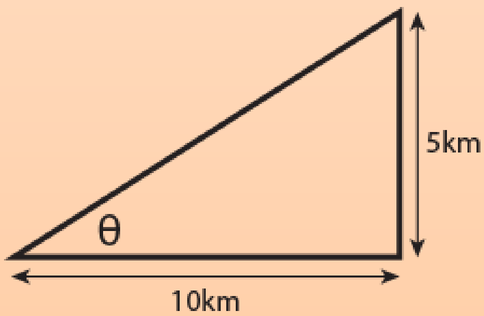


You will, quite rightly, have worked out that it's not quite as simple as all that, because the actual direction of travel depends on the tide speed and your speed, but you can probably see why trigonometry might be important!

Example

You are out for a day's sailing, and don't really mind where you end up. You started out heading due east, and plan to sail for one hour at a cruising speed of 10 km/h. The tide is due north, and running at 5km/h. What direction will you end up travelling in?

1. **First draw your triangle**, and label the sides. You are heading due east, so let's make that the bottom of the triangle, length 10km. The tide is going to push you north, so let's make that the right hand side. And you want to know what direction you'll end up going in, so that's angle θ.

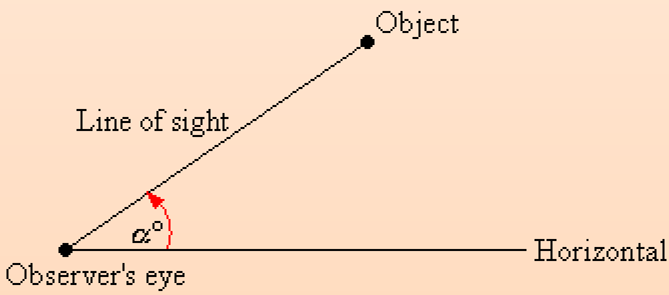


2. **You have the opposite and the adjacent, which means that you need to use tan.** Tan θ = Opposite/adjacent $= \frac{5}{10} = 0.5$.
3. **Now is the time to use the inverse tan function.** The inverse tan of 0.5 is 26.6°. In other words, tan 26.6 = 0.5.
4. **Direction is measured from North**, which is 0°. Your answer from (3), however, is measured from 90°, or East. You will therefore need to subtract your answer from 90°, to obtain the answer that you are travelling in a direction of 63.4°, which is between North East (45°) and East North East (67.5°).

Why is this important? You'll need to know which direction you travelled in order to sail home, of course!

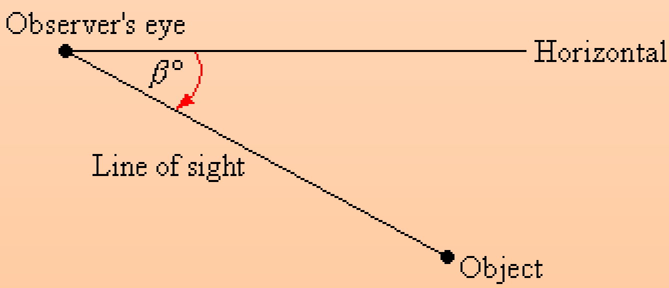
Angles of Elevation and Depression

The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).



The angle of elevation of the object from the observer is $\alpha°$.

If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression.



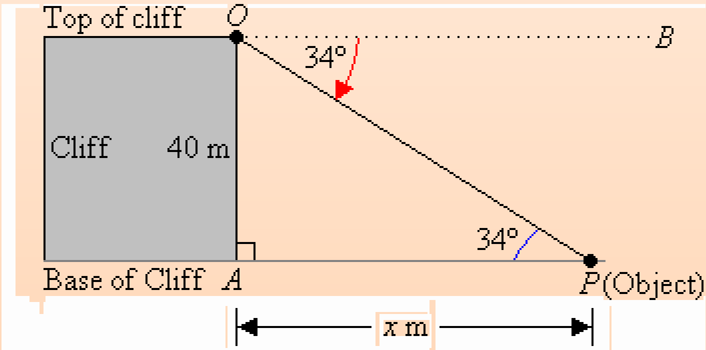
The angle of depression of the object from the observer is $\beta°$.

Example

From the top of a vertical cliff 40 m high, the angle of depression of an object that is level with the base of the cliff is 34º. How far is the object from the base of the cliff?

Solution

Let x m be the distance of the object from the base of the cliff.



Angle of depression $= 34°$

But $\angle APO = \angle BOP$ {Alternate angles}

$\therefore \angle APO = 34°$

From ΔAPO, we have:

$\tan 34° = \dfrac{40}{x}$ {TOA}

$\therefore x \tan 34° = \dfrac{40}{x} \times x$ {Multiply both sides by x}

$0.6745x = 40$ {Divide both sides by 0.6745}

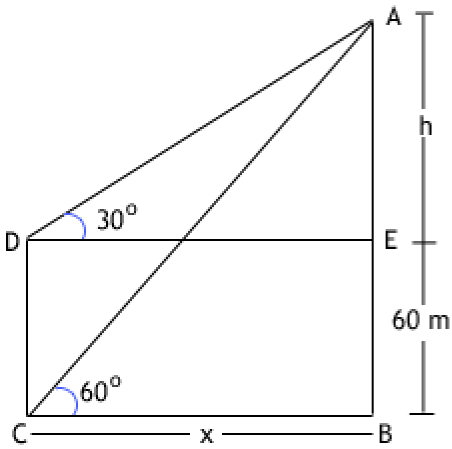
$\dfrac{0.6745x}{0.6745} = \dfrac{40}{0.6745}$

$x = 59.30$

So, the object is 59.30 m from the base of the cliff.

Exercise 15

1. The angles of elevation of the top of a tower from the top and bottom of a 60 m high building are 30° and 60°. What is the height of the tower?



2. Achan's kite is flying above a field at the end of 65 m of string. If the angle of elevation to the kite measures 70°, how high is the kite above Brian's head?
3. From an airplane at an altitude of 1200 m, the angle of depression to a building on the ground measures 28°. Find the distance from the plane to the building.
4. From a point on the ground 12 ft. from the base of a flagpole, the angle of elevation of the top of the pole measures 53°. How tall is the flagpole?
5. From a plane flying due east at 265 m above sea level, the angles of depression of two ships sailing due east measure 35° and 25 °. How far apart are the ships?
6. A man flies a kite and lets out 100 feet of string. The angle of elevation of the string is 52°. How high off the ground is the kite? How far away is the man from the spot directly under the kite?
7. From the top of a vertical cliff 40 m high, the angle of depression of an object that is level with the base of the cliff is 34º. How far is the object from the base of the cliff?
8. An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant until the airplane flies over the building.

9. A 14 foot ladder is used to scale a 13 foot wall. At what angle of elevation must the ladder be situated in order to reach the top of the wall?
10. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area?
11. One of the legs of a right triangle is twice as long as the other, and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.
12. Standing on a cliff 380 meters above the sea, Pat sees an approaching ship and measures its angle of depression, obtaining 9 degrees. How far from shore is the ship? Now Pat sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?
13. In a right triangle, the 58-cm hypotenuse makes a 51-degree angle with one of the legs. To the nearest tenth of a cm, how long is that leg?

UNIT 4: ALGEBRA

EXPRESSIONS, EQUATIONS AND INEQUALITIES

Investigation: Work in groups.

Find out as much as you can about the early history of algebra. You might consider:

a) Ahmes Papyrus (Egyptian c. 1700 BC)
b) Diophantus (Greek c. AD 250)
c) Mohammed ibn Musa al-Khowarizmi (Arab c. AD 825)
d) Bhaskara (Hindu c. AD 1150)

Present your findings to the whole class.

Simplification of expressions

In algebra we are doing arithmetic with just one new feature, we use letters to represent numbers. Because letters are simply stand-ins for numbers, arithmetic is carried out exactly as it is with numbers.

Algebraic expressions are general arithmetic statements where letters are used to represent numbers. For instance

'$x+5$' means 'add 5 to a variable x' or' 5 more than x'

'$a+b$' means 'the sum of a and b'

We can also write a statement in form of symbols, for example,

'the product of a and b' can be written as 'ab'

c is y subtracted from x is $c=x-y$

Exercise 1: In groups,

1. Write in words, the meaning of:
   a) $c-b$
   b) $2a+d$

c) $3(l+m)$

d) p^2+q

e) $a+b$

2. Write the following as an algebraic expression:
   a) The sum of p and the square of q
   b) three times the product of a and b
   c) twice n minus m
   d) The square of the sum of c and d
   e) 10 less than c
3. Write in words the meaning of:
   a) $A = 2x + 2y$
   b) $X = \frac{2c}{d}$
   c) $B = 3a^2 + d$
   d) $f = 4x - y$

Simplifying algebraic expressions

Before you evaluate an algebraic expression, you need to simplify it. This will make all your calculations much easier. Here are the basic steps to follow to simplify an algebraic expression:

1. remove parentheses by multiplying factors
2. use exponent rules to remove parentheses in terms with exponents
3. combine like terms by adding coefficients
4. combine the constants

Let's work through an example.

$$5(2+x)+3(5x+4)-(x^2)^2$$

When simplifying an expression, the first thing to look for is whether you can clear any parentheses. Often, you can use the distributive property to clear parentheses, by multiplying the factors times the terms inside the parentheses. In this expression, we can use the distributive property to get rid of the first two sets of parentheses.

$$= 10 + 5x + 15x + 12 - (x^2)^2$$

When a term with an exponent is raised to a power, we multiply the exponents, so $(x^2)^2$ becomes x^4.

$$= 10 + 5x + 15x + 12 - x^4$$

The next step in simplifying is to look for like terms and combine them. The terms $5x$ and $15x$ are like terms, because they have the same variable raised to the same power -- namely, the first power, since the exponent is understood to be 1. We can combine these two terms to get $20x$.

$$= 10 + 20x + 12 - x^4$$

Finally, we look for any constants that we can combine. Here, we have the constants 10 and 12. We can combine them to get 22.

$$= 22 + 20x - x^4$$

Now our expression is simplified. Just one more thing -- usually we write an algebraic expression in a certain order. We start with the terms that have the largest exponents and work our way down to the constants. Using the commutative property of addition, we can rearrange the terms and put this expression in correct order, like this.

$$= -x^4 + 20x + 22$$

Exercise 2: Simplifying Algebraic expressions

In groups, write the following statements as algebraic expressions and simplify where necessary.

Simplify each of the following by collecting like terms:

(a) $4a + b + 2a$

(b) $4b + 2c + 6b + 3c$

(c) $4a + 5b - a + 2b$

(d) $14p + 11q - 8p + 3q$

(e) $6x - 4y + 8x + 9y$

(f) $11x + 8y + 3z - 2y + 4z$

(g) $16x - 8y - 3x - 4y$

(h) $11y + 12z - 10y + 4z + 2y$

Algebraic Substitution

In Algebra "Substitution" means putting numbers where the letters are:

	When we have:	$x - 2$
	And we know that x=6 ...	
	... then we can "substitute" **6** for **x**:	$6 - 2 = 4$

Example 1: When x=2, what is $\frac{10}{x}$+ 4 ?

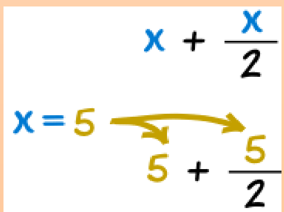
Put "2" where "x" is:

$$\frac{10}{2} + 4 = 5 + 4 = \mathbf{9}$$

Example 2: When x=5, what is x + $\frac{x}{2}$+?

Put "5" where "x" is:

$5 + \frac{5}{2} = 5 + 2.5 = \mathbf{7.5}$



Example 3: If x=3 and y=4, then what is x^2 + xy ?

Put "3" where "x" is, and "4" where "y" is:

$$3^2 + 3\times4 = 3\times3 + 12 = \mathbf{21}$$

Example 4: If x=3 (but we don't know "y"), then what is x^2 + xy ?

Put "3" where "x" is:

$$3^2 + 3y = \mathbf{9 + 3y}$$

(that is as far as we can get)

As that last example showed, we may not always get a number for an answer, sometimes just a simpler formula.

Negative Numbers

When substituting negative numbers, put ( ) around them so you get the calculations right.

Example 1: If $x = -2$, then what is $1 - x + x^2$?

Put "(−2)" where "x" is:

$$\mathbf{1 - (-2) + (-2)^2} = 1 + 2 + 4 = 7$$

In that last example:

- the $\mathbf{-(-2)}$ became $\mathbf{+2}$
- the $\mathbf{(-2)^2}$ became $\mathbf{+4}$

Exercise 3

Working in pairs, answer the following questions:

1. Given that $p = 2z - m$, find
   i) p when $z = 10$ and $m = 2$
   ii) p when $z = -5$ and $m = 3$
   iii) m when $p = 20$ and $z = 2$

2. If $p = 4, q = -2$ and $r = 3$, find the value of

i) $3q - 2r$

ii) $2pq - r$

iii) $\dfrac{p - 2q + 2r}{p + r}$

3. If $a = 3, b = -2$ and $c = -1$, find the value of

i) b^2

ii) $ab - c^3$

4. If $p = 4, q = -3$ and $r = 2$, find the value of

i) $\sqrt{p - q + r}$

ii) $\sqrt{p^2 + q^2}$

5. A formula states:

$y = 4x - 5$

a) Calculate y if $x = 3$.

b) Determine x if $y = 23$.

c) Determine x if $y = 8$.

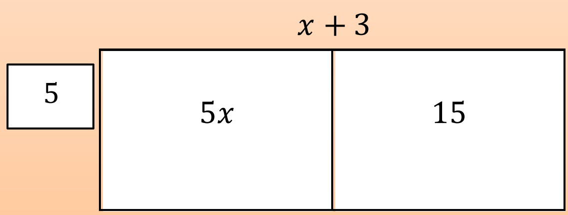
Expansion of algebraic expressions

Expanding is to remove brackets from an algebraic expression.

One bracket

All terms inside the bracket are multiplied by the term outside.

$$5(x + 3) = 5x + 15$$



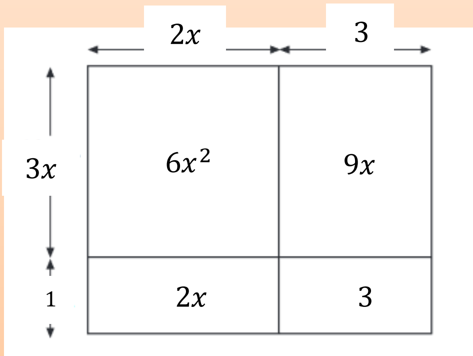
Note that the negative sign belongs to the term directly following it.

e.g. -3(4a – 5) = $(-3) \times 4a + (-3) \times (-5) = -12a + 15$

Multiplying out two brackets

If there are two brackets, multiply everything in the first bracket by everything in the second bracket. To simplify, add the like terms together.

$$(2x+3)(3x+1) = 6x^2 + 9x + 2x + 3 = 6x^2 + 11x + 3$$



Brackets and squares

If a bracket is squared, write it as bracket multiplied by bracket and expand.

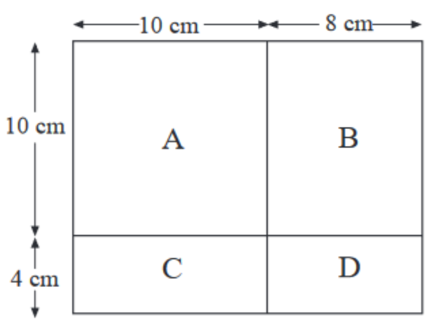
e.g. $(2x+3)^2 = (2x+3)(2x+3)$

$= 4x^2 + 6x + 6x + 9$

$= 4x^2 + 12x + 9$

Exercise 4: In pairs, work out the following questions.

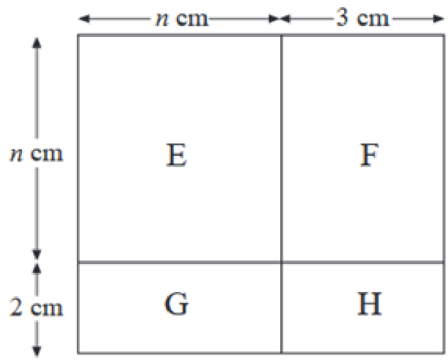
1. a) The diagram shows a rectangle 18 cm long and 14 cm wide. It has been split into four smaller rectangles, A, B, C and D.
    i) Write down the area of each of the small rectangles. One has been done for you.



Area of Rectangle C = 40 cm^2.

ii) What is the area of the *whole* rectangle?
iii) What is 18 x 14?

a) The diagram shows a rectangle $(n+3)$ cm long and $(n+2)$ cm wide. It has been split into four smaller rectangles.
a) Write down a number or an expression for the area of each small rectangle. One has been done for you.



Area of Rectangle F = $3n$ cm^2.

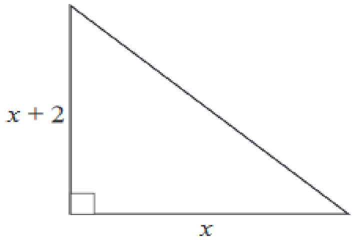
iii) What is $(n+3)(n+2)$ multiplied out?

2. Expand:

a. $(x + 6)$
b. $3 (x - 4)$
c. $5 (2x + 6)$
d. $7 (3x - 4)$
e. $3 (2x + 4)$
f. $8 (3x - 9)$
g. $(-2)(x - 4)$
h. $(-3)(8 - 2x)$
i. $5 (3x - 4)$
j. $9 (2x + 8)$

3. Deng writes $3 (4x - 8) = 12x - 8$. Explain why his expansion is *not* correct.

4. Write down an expression for the area of this triangle, that:
a) contains brackets,
b) does *not* contain brackets.



5. Expand each of the following expressions by multiplying out the brackets.

   a) $(x+4)(x+3)$ c) $(x+2)(x+4)$ e) $(x+1)(x+5)$

   b) $(x+6)(x-1)$ d) $(x-4)(x+2)$ f) $(x-3)(x+2)$

6. Simplify each of the following expressions.
   a) $(x+2)(x+4)+(x+1)(x+2)$
   b) $(x+3)(x+7)+(x-1)(x+5)$
   c) $(x+6)(x+2)-(x-2)(x+3)$
   d) $(x-4)(x-8)-(x-1)(x-9)$

7. Expand.
   a) $(x+1)^2$
   b) $(x-2)^2$
   c) $(x+3)^2$
   d) $(x+5)^2$
   e) $(x-7)^2$
   f) $(x-8)^2$
   g) $(x+\ 10)^2$
   h) $(x-12)^2$
   i) $(x+4)^2$
   j) $(2x+3)^2$
   k) $(4x-7)$
   l) $(3x+2)^2$
   m) $(4x+1)^2$
   n) $(5x-2)^2$
   o) $(6x-4)^2$

8. Expand:

   b) $(x+1)^3$ b) $(2x+1)^3$ c) $(x-5)^3$

Factorizing algebraic expressions

In this section, we consider examples of the process of factorizing, whereby the process of removing the brackets is reversed and introduced into expressions.

Example 1

Factorise:

a) $8x + 12$ b) $35x + 28$

Solution

a) Note that both terms are multiples of 4, so we can write,

$$8x + 12 = 4\,(2x + 3)$$

b) Here both terms are multiples of 7, so

$$35x + 28 = 7(5x + 4)$$

Results like these can be checked by multiplying out the bracket to get back to the original expression.

Example 2

Factorise,

a) $x^2 + 2x$ b) $3x^2 - 9x$ c) $x^3 - x^2$

Solution

a) Here, as both terms are multiples of x, we can write,

$$x^2 + 2x = x(x + 2)$$

b) In this case, both terms are multiples of x and 3, giving,

$$3x^2 - 9x = 3x(x - 3)$$

c) In this example, both terms are multiples of x^2.

$$x^3 - x^2 = x^2(x - 1)$$

Example 3

Factorise:

a) $x^2 + 9x + 18$ b) $x^2 + 2x - 15$ c) $x^2 - 7x + 12$

Solution

a) This expression will need to be factorized into two brackets.

$$x^2 + 9x + 18 = (x \quad)(x \quad)$$

As the expression begins x^2, both brackets must begin with x. The two numbers to go into the brackets must multiply together to give 18 and add to give 9. So they must be 3 and 6, giving,

$$x^2 + 9x + 18 = (x + 3)(x + 6)$$

You can check this result by multiplying out the brackets.

b) We note first that two brackets are needed and that both must contain an x as shown

$$x^2 + 2x - 15 = (x \quad)(x \quad)$$

Two other numbers are needed which, when multiplied give -15 and when added give 2. In this case, these are -3 and 5. So the factorization is,

$$x^2 + 2x - 15 = (x - 3)(x + 5)$$

Check the result by multiplying out the bracket.

c) Again, we begin by noting that,

$$x^2 - 7x + 12 = (x \quad)(x \quad)$$

We require two numbers which, when multiplied give 12 and when added give -7. In this case, these numbers are -3 and -4.

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Exercise 5

1. Factorise:

a) $4x - 2$
b) $6x - 12$
c) $5x - 20$
d) $4x + 32$
e) $6x - 8$
f) $8 - 12x$
g) $21x - 14$
h) $15x + 20$
i) $30 - 10x$

2. Factorise:

a) $x^2 + 4x$
b) $x^2 - 3x$

c) $4x - x^2$
d) $6x^2 + 8x$
e) $9x^2 + 15x$
f) $7x^2 - 21x$
g) $28x - 35x^2$
h) $6x^2 - 14$
i) $5x^2 - 3x$

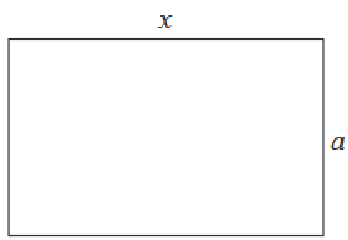
3. Factorise:
   i) $x^3 + x^2$
   ii) $2x^2 - x^3$
   iii) $4x^3 - 2x^2$
   iv) $8x^3 + 4x^2$
   v) $16x^2 - 36^3$
   vi) $4x^3 + 22x^2$
   vii) $16x^2 - 6x^3$
   viii) $14x^3 + 21x^2$
   ix) $28x^3 - 49x^2$

4. a) Expand $(x+5)(x-5)$
   b) Factorise $x^2 - 25$
   c) Factorise each of the following.
      i. $x^2 - 49$
      ii. $x^2 - 64$
      iii. $x^2 - 100$
      ii. $x^2 - a^2$
      iii. $x^2 - 4b^2$

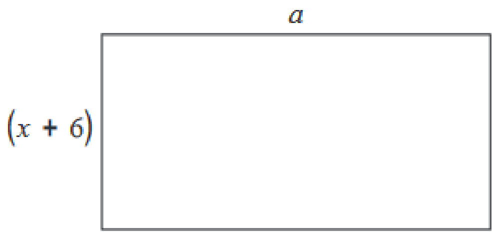
5. Factorise:
   a) $x^2 + 7x + 12$
   b) $x^2 + 8x + 7$
   c) $x^2 + 11x + 18$
   d) $x^2 + 12x + 27$
   e) $x^2 + 17x + 70$
   f) $x^2 + 6x + 8$

Extension questions

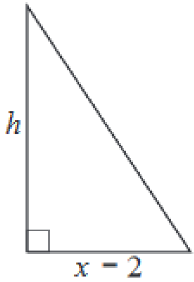
6. The area of the rectangle shown is $x^2 - 5x$. Express a in terms of x.



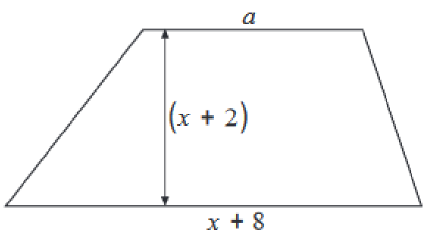
7. The area of the rectangle shown is $x^2 + 11x + 30$. Express a in terms of x.



8. The area of the triangle shown is $\frac{1}{2}x^2 + \frac{3}{2}x - 5$. Express h in terms of x.



9. The area of the trapezium shown is $\frac{1}{2}x^2 + 10x + 18$. Determine a.



Solving Equations

Algebraic equations are equations which contain at least one unkown or variable. $4x - 7 = x + 5$ is an algebraic equation.

It has a left hand side (LHS) and a right hand side (RHS) separated by an equal sign.

$$\underbrace{4x - 7}_{\text{LHS}} = \underbrace{x + 5}_{\text{RHS}}$$

The solution of an equation is the value or values of the variable which make the equation true.

When the unknown is on both sides, we need to rearrange the equation so that the unknown is on one side and the numbers are on the other side of the equals sign. We do this by doing the same thing to both sides of the equation to keep the equals sign true.

For the equation $4x - 7 = x + 5$, subtract x from both sides

$$4x - 7 - x = x + 5 - x$$

$$3x - 7 = 5$$

Add 7 to both sides

$$3x - 7 + 7 = 5 + 7$$

$$3x = 12$$

Divide both sides by 3

$$x = 4$$

To check our solution when $x = 4$,

the LHS $= 4 \times 4 - 7 = 16 - 7 = 9$ and the RHS $= (4) + 5 = 9$

So, the solution of the equation is $x = 4$.

Simultaneous Equations

Nyoka buys a coffee and cake for SSP. 50.
Othow buys two coffees and a cake for SSP 85.
How much does the coffee cost?

You can figure out that the coffee must cost SSP 35. We can use mathematics to solve problems like these.

So, let the cost of the coffee be x, and the cost of the cake be y.
For Nyoka, $x + y = 50$
For Othow $2x + y = 85$
These equations can be subtracted

$$2x + y - (x + y) = 85 - 50$$
$$x = 35$$

We can find y by substituting for x in the equation for Nyoka or Othow.

$$35 + y = 50$$
$$y = 50 - 35 = 15$$

The solution to these equations is $x = 35, y = 15$.
We can check our solution by substituting in the equation for Othow.

$$2x + y = 2 \times 35 + 15 = 85$$

Example

Solve the simultaneous equations
$4x + 5y = 17$
$3x + y = 10$

Solution

The equation $3x + y = 10$ can be rearranged to give, $y = 10 - 3x$.

This can be substituted into the equation $4x + 5y = 17$ to give

$$4x + 5(10 - 3x) = 17$$

Expanding the brackets, $4x + 50 - 15x = 17$

Simplifying, $50 - 11x = 17$

Add $11x$ to both sides, $50 = 17 + 11x$

Subtract 17 from both sides, $33 = 11x$

Divide both sides by 11, $x = 3$

Substituting for x in equation $y = 10 - 3x$, $y = 10 - 3 \times 3 = 1$

The solution to these equations is $x = 3, y = 1$

We can check by substituting in the original equations, e.g.
$4x + 5y = 4 \times 3 + 5 \times 1 = 12 + 5 = 17$

Exercise 6

1. Solve the following equations.

a) $2x - y = 1$
$2x + 2y = 10$

b) $2x + 3y = 8$
$5x - 2y = 1$

c) $3x + 3y = 7$
$4x + 3y = 2$

d) $5x - y = 15$
$x + y = -3$

e) $3x - y = 6$
$2x + 5y = -13$

f) $3x + 5y = 1$
$x - y = -5$

g) $2x - 5y = 3$
$3x + 2y = 14$

h) $2x + 3y = 6$
$3x + 2y = -1$

i) $3x - y = 18$
$x + 2y = -1$

2. Make up some equations of your own for your friends to try.

FORMATION AND SOLUTION OF INEQUALITIES

Equations and inequalities are both mathematical sentences formed by relating two expressions to each other. In an equation the two expressions are deemed equal which is shown by the symbol =.

$$x = y \qquad x \text{ is equal to } y$$

Where as in an inequality the two expressions are not necessarily equal which is shown by the symbols: $>, <, \leq \ or \ \geq$.

$x > y$	x is greater than y
$x \geq y$	x is greater than or equal to y
$x < y$	x is less than y
$x \leq y$	x is less than or equal to y

An equation or an inequality that contains at least one variable is called an open sentence.

When you substitute the variable in an open sentence with a number the resulting statement is either true or false. If the statement is true the number is a solution to the equation or inequality.

Example 1

Is 3 a solution to

$$5x + 14 = 245x + 14 = 24$$

Substitute x for 3

$$5 \times 3 + 145 \times 3 + 14$$

$$15 + 14 = 29 \neq 2415 + 14 = 29 \neq 24$$

FALSE!
Since 29 is not equal to 24, 3 is not a solution to the equation.

Example 2
Solve the inequality $2x < 5$.
2 times x is less than 5.

Divide both sides by 2.

x is less than $\frac{5}{2}$.

So the solution is:

$x < \frac{5}{2}$.

So x is any number less than $\frac{5}{2}$.

Example 3

Solve the inequality $3x + 1 > 7$.

We want the x term on the left-hand side, by itself.

So, take 1 away from the left-hand side.

To balance the equation, we must do the same to the right hand side.

So, take 1 away from the right-hand side.

This gives us, $3x + 1 - 1 > 7 - 1$

So, $3x > 6$.

$$x > \frac{6}{3} = 2$$

The solution is $x > 2$.

Example 4

Solve the inequality $4a - 2 < 5$.

Add 2 to both sides, to leave $4a$ on the left, by itself:

$4a - 2 + 2 < 5 + 2$

So, $4a < 7$.

Therefore, the solution is $a = \frac{7}{4}$.

Example 5

Solve the inequality $2(y + 5) < 16$.

Either:

$2y + 10 < 16$ (multiplying out the brackets.)

So, $2y + 10 - 10 < 16 - 10$ (subtracting 10 from both sides.)

$2y = 6$

Therefore, $y = 3$.

Or:
Divide both sides of $2(y+5) < 16$ by 2, to get
$y + 5 < 8$.
Therefore $y = 3$

Exercise 7

Solve the following inequalities.

1. $4a < 9$
2. $2y + 6 > 14$
3. $3x - 5 < 1$
4. $3(4 + x) < 15$
5. $4x + 20 < 100$
6. $10x + 16 > 156$
7. $25 + 6x < 300$
8. $-5x > 80$

UNIT 5: STATISTICS & PROBABILITY

Every day we receive a lot of information.

For example:

- Lucy scores 55% in her mathematics exam.
- There are 20 000 vehicles in the city every day.
- The month of July 2017 was the coldest month in 5 years.

Some statements cannot be made without gathering information over a long period of time. This information must be collected and analysed accurately.

The facts or pieces of information that are collected are called **data**. **Data** may be collected by counting, measuring or asking questions.

Statistics is the collection, analysis, interpretation and presentation of data.

Types of data:

a) Discrete numerical data b) Continuous data c) Categorical data

Discrete numerical data: This is data made up of variables that can be **quantified or counted** e.g. the number of students who like math, number of cars in the town or the scores for a test.

We organize discrete data using a **frequency table**. The data can be displayed by a **bar chart.**.

The Mean, Mode and Median

There are three different values which are commonly used to measure the **central tendency or location** of a set of numerical data. These are the **mean or average**, the **median** and the **mode**.

THE MEAN

The **mean** or **average** is the total of all data values divided by the number of data values.

We use the symbol $\bar{x}$ to represent the mean.

$$\bar{x} = \frac{sum\ of\ data\ values}{number\ of\ data\ values}$$

THE MEDIAN

The **median** of a set of data is the middle value of the ordered set. For an odd number of data values there is a middle value which is the median.

For an even number of data values there are two middle values. The median is the average of these two values.

THE MODE

The **mode** is the value which occurs most often in a data set.

Example 1

A tennis player has won the following matches in tournaments during the last 18 months.

1 2 0 1 3 1 4 2 1 2 3 4 0 0 1 2 2 3 2 1 6 3 2 1

1 1 1 2 2 0 3 4 1 1 2 3 0 2 3 1 4 1 2 0 3 1 2 1

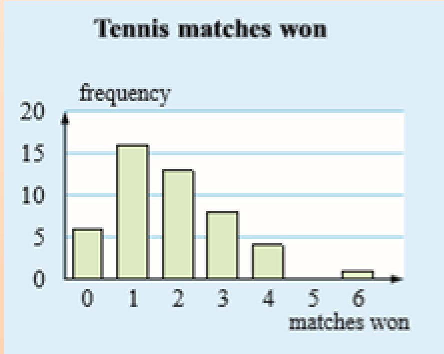
a) Organise the data to form a frequency table.
b) Draw a bar chart
c) How many times did the player advance past the second match of a tournament?
d) On what percentage of occasions did the player win less than 2 matches?

Solution:

a)

Wins	*Frequency*
0	*6*
1	*16*
2	*13*
3	*8*
4	*4*
5	*0*
6	*1*
	48

b)



c) 13+8+4+1=26 times

d) 6+16=22 occasions

$$\text{Therefore, percentage} = \frac{22}{48} \times 100 = 45.8\%$$

Investigative Task:

Work in groups to undertake the following tasks. Each group should be assigned one task which they will present after completion.

Task 1: To test the theory that "many students do not like mathematics".

Task 2: To test the theory that "the most commonly used word in newspaper articles is 'the'.

Task 3: To test the theory that "the most watched sport is football".

What to do:

1. State the chosen for collecting your data and explain why you chosen this method.

2. Organize your data in a frequency table.
3. Find the mean, mode and median for your data. Explain which of the three values best summarizes your data.
4. Analyze the truth of the theory being tested using your data.
5. Submit a clear report on your finding to your teacher clearly explaining how effective your data collection and organization was?

Example 2

An exceptional footballer scores the following numbers of goals for her school during a season:

1 3 2 0 4 2 1 4 2 3 0 3 3 2 2 5 2 3 1 2

Find the:

a) mean b) median c) mode for the number of goals she scored.

Solution

a) Mean$=\frac{sum\ of\ all\ scores}{number\ of\ matches} = \frac{45}{20} = 2.25\ goals$

b) The ordered data is

0 0 1 1 1 2 2 2 2 2 2 2 3 3 3 3 3 4 4 5

There are two values in the middle: 2 2

Median$=\frac{2+2}{2} = 2\ goals$

c) Mode = 2 goals (2 occurs most frequent)

Exercise 1

1. Find the mean, mode and median of the following data sets:
a) 3, 2, 2, 5, 4, 4, 3, 2, 6, 4, 5, 4, 1
b) 7, 8, 0, 3, 0, 6, 0, 11, 1

2. Consider performances of two groups of students in the same mental arithmetic test out 10 marks.

   Group X: 7, 6, 6, 8, 6, 9, 7, 5, 4, 7

   Group Y: 9, 6, 7, 6, 8, 10, 3, 9, 9, 8, 9

   a) Calculate the mean for each group
   b) There are 10 students in group *X* and 11 students in group *Y*. Is it unfair to compare the mean scores for these groups?
   c) Which group performed better at the test?

b) **Continuous numerical data:** This is data that can take any value on a number line. A continuous variable has to be measured. e.g. heights of students, speed of cars on the highway.

When data is recorded for a continuous variable, there are likely to be many different values. This data is therefore organized by grouping it into **class intervals**.

A **frequency histogram** is used to display the data. It is similar to a bar chart but because the data is continuous, there is no space between the columns and the horizontal axis is a number line.

Example:

The weight of pumpkins harvested by Susan from her garden was recorded in kilograms:

2.1, 3.0, 0.6, 1.5, 1.9, 2.4, 3.2, 4.2, 2.6, 3.1, 1.8, 1.7, 3.9, 2.4, 0.3, 1.5, 1.2

Organise the data using a frequency table, hence graph the data.

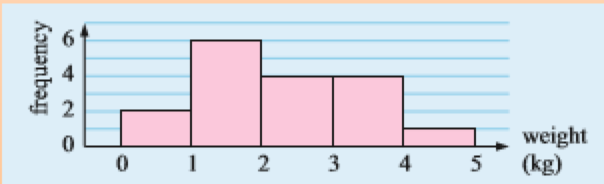
Solution:

The data is continuous because the weight could be any value from 0.1kg to 10kg. The lowest weight recorded was 0.3kg and the heaviest was 4.2kg, so we can divide the data into class intervals of 1kg.

The class interval $1 \leq w < 2$ includes all weights from 1 kg up to, but not including 2 kg.

Weight (kg)	*Frequency*
$0 \leq w < 1$	2
$1 \leq w < 2$	6
$2 \leq w < 3$	4
$3 \leq w < 4$	4
$4 \leq w < 5$	1

The frequency histogram is shown below:



From the frequency graph and the frequency table, it is easy to tell which class has the highest frequency. We call this class the **modal class**.

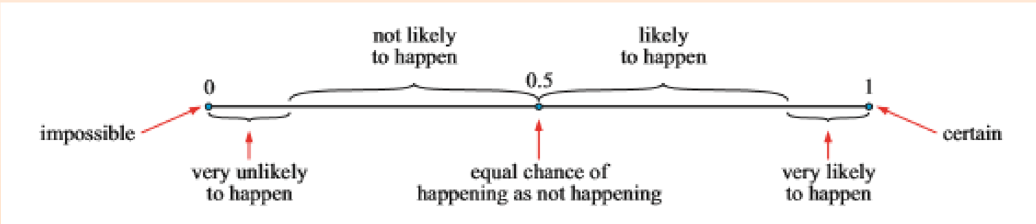
In the example above, $1 \leq w < 2$ is the modal class.

Probability

The study of probability deals with the **chance** or **likelihood** of an event happening. For every event we can carefully assign a number which lies between 0 and 1 inclusive.

- An impossible event which has 0% chance of happening is assigned a probability of 0.
- A certain event which has a 100% chance of happening is assigned a probability of 1.
- All other events between these two extremes can be assigned a probability between 0 and 1.

The number line below shows how we could interpret different probabilities:



Discussion

3. In pairs, discuss these words.

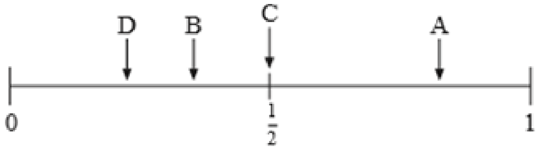
a) Likely	g) Impossible
b) Might	h) May
c) Certain	i) Hardly likely
d) Possible	j) Very unlikely
e) Probable	k) Poor chance
f) Even chance	l) Unlikely

2. Write down 12 different sentences about real life events. Each sentence should contain one of these words above. Share your sentences with the rest of the class. Does everyone agree with them?

Discussion

1. The events A, B, C and D have probabilities as shown on this probability line:

   a) Which event is most likely to take place?



   b) Which events are more likely to take place than event B?

   c) Which events are less likely to take place than event A?

Task: Complete in groups.

1. Toss a coin 100 times recording the number of times it fall on heads or tails in your exercise books. Find the experimental probability that it falls on:
    a) heads
    b) tails
2. Toss two coins at the same time 200 times recording the number of times you obtain:

    i) two heads
    ii) two tails
    iii) head and tail
    Find the experimental probability of obtaining each of the results above?

We usually assign either;

- An **experimental** probability by observing the results of an experiment
- A **theoretical** probability by using arguments of symmetry.

In general, the probability of an event occurring is

$$P\ (event) = \frac{the\ number\ of\ ways\ the\ event\ can\ happen}{total\ number\ of\ outcomes}$$

Experimental Probability

The following terms are used in experimental probability:

- The **number of trials** is the total number of times the experiment is repeated.
- The **outcomes** are the different results possible for one trial of the experiment.
- The **frequency** of a particular outcome is the number of times that outcome is observed.
- The **relative frequency** of an outcome is the frequency of that outcome expressed as a fraction or percentage of the total number of trials.

Exercise 2

1. When John makes an error in his work he crushes the sheet of paper into a ball and throws it at the waste basket. At the end of the day he has scored 16 hits into the basket and made 5 misses. Find the experimental probability that he scores a hit into the basket.

2. In the first round of competition Sasha recorded 77 hits out of 80 shots at her target.
   a) Use this result to estimate her chances of hitting the target
   b) On the next day in the final round of the competition, she scored105 hits out of 120 shots. Obtain the 'best 'estimate of her hitting the target with any shot.

   (best estimate is found by adding the numbers of hits and dividing the result by the total number of shots)

3. Paulo catches a 7:45 am bus to school. During a period of 79 days he arrives at school on time on 53 occasions. Estimate the probability that Paulo:
   a) arrives on time.
   b) arrives late.

4. Don threw a tin can into the air 180 times. From these trials it landed on its side 137 times. Later that afternoon he threw the same tin can into the air 150 times. It landed on its side 92 times.
   a) Find the experimental probability of 'landing on the side' for both sets of trials.
   b) List possible reasons for the differences in the results.

5. In a large county hospital, 72 girls and 61 boys were born in a month.
   a) How many children were born in total?
   b) Estimate the probability that the next baby born at this hospital will be:

      i) a girl ii) a boy

Sample space:

The **outcomes** of an experiment are the different possible results we could obtain in one trial.
The **sample space** of an experiment is the set of its possible outcomes.
Suppose we take 10 cards with the numbers 1 to 10 written on them. If we take a card at random, it have any of the numbers from 1 to 10 on it.
The outcomes are the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 or 10.
The sample space is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
The number of elements in the sample space is 10.

Discussion questions

1. List the sample space for:
   a) rolling a six-sided die
   b) taking a card at random from a pack of playing cards and looking at its suit
   c) choosing at random a day of the week
   d) twirling a spinner with 6 segments marked A, B, C, D, E and F.

2. We have 12 cards which have the numbers from 1 to 12 written on them. A card is taken at random. Find the number of elements in the sample space.
   a) Event A is 'getting an even number'.
      i) List the outcomes in the event
      ii) Give the number of outcomes in the event.
   b) Event B is 'getting a number less than 5'
      i) List the outcomes in the event
      ii) Give the number of outcomes in the event.
   c) Event C is 'getting a multiple of 4'
      i) List the outcomes in the event
      ii) Give the number of outcomes in the event.

Theoretical probability

Consider an unbiased coin. If the coin is tossed, there is an equal likelihood of obtaining heads or tails. We therefore say that the likelihood of obtaining heads is 1 chance out of 2.

The theoretical probability of a particular event is the theoretical chance of that event occurring in any trial of the experiment.

In general, for an event E containing **equally likely** possible results:

$$p(E) = \frac{the\ number\ of\ outcomes\ in\ the\ event\ E}{the\ total\ number\ of\ possible\ outcomes}$$

The Principle of exhaustion

In a particular context with a set of exclusive outcomes (e.g. A, B, C), the sum of the probabilities is always 1.

$$p(A) + p(B) + p(C) = 1$$

Example:

A ticket is randomly selected from a basket containing 3 green, 4 yellow and 5 blue tickets. Determine the probability of getting:

a) a green ticket

b) a green or yellow ticket

c) an orange ticket

d) a green, yellow or blue ticket

Solution:

a) p(green) $= \frac{3}{12} = \frac{1}{4}$

b) p(a green or a yellow) $= \frac{3+4}{12} = \frac{7}{12}$

c) p(orange) $= \frac{0}{12} = 0$

d) p(green, yellow or blue) $= \frac{3+4+5}{12} = \frac{12}{12} = 1$

Complementary events:

If E is an event, then E' is the complementary event of E, and

P(E) + P(E')=1 or P(E not occurring) = 1 – P(E occurring)

Example:

An ordinary 6 sided die is rolled once. Determine the chance of

a) getting a 6 b) not getting a 6

c) getting a 1 or 2 d) not getting a 1 or 2

Solution:

The sample space of possible outcomes is {1, 2, 3, 4, 5, 6}

a) P(6) $=\frac{1}{6}$ b) P(not getting a 6)$=\frac{5}{6}$

c) P(1 or 2) $=\frac{2}{6}$ d) P(not getting a 1 or 2)$=\frac{4}{6}$

Exercise 3

1. A marble is selected randomly selected from a box containing 5 green, 3 red and 7 blue marbles. Determine the probability that the marble is:

a) red
b) green
c) blue
d) not red
e) neither green nor blue
f) green or red

2. A carton contains eight brown and four white eggs. Find the probability that an egg selected at random is:

a) brown
b) white

3. In a class of 32 students, eight have one first name, nineteen have two first names, and five have three first names. A student is selected at random. Determine the probability that the student has:

a) no first name
b) one first name
c) two first names

d) three first names.

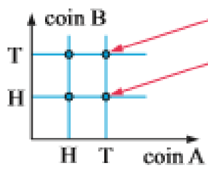
4. An ordinary six sided doe is rolled once. Determine the chance of getting:
   a) a 5
   b) an odd number
   c) a number greater than 1
   d) a multiple of 2
5. In a club newsletter, 8 pages contain reports, 3 pages contain articles and 5 pages contain advertising. The newsletter is opened to a page at random. Determine the probability that it is:
   a) a report
   b) advertising
   c) not advertising
   d) a report or articles

Using grids to find probabilities

When an experiment involves more than one operation we can still list the sample space.

If we have 2 operations we can use a two dimensional grid (sample space diagram) to illustrate the sample space efficiently.

The grid shown below shows the outcomes when 2 coins A and B are tossed:



This point represents 'a tail from coin A' and 'a tail from coin B'.

This point represents 'a tail from coin A' and 'a head from coin B'.

There are four members of the sample space.

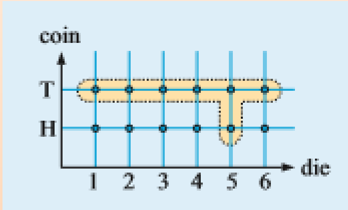
They are: HH, HT, TH, and TT.

Example:

Use a two dimensional diagram to illustrate the sample space for tossing a coin and roling a die simultaneously. From this grid determine the probability of:

a) tossing a head
b) rolling a 2
c) getting a tail and a 5
d) getting a tail or a five

There are 12 outcomes in the sample space



Or

Coin/die	1	2	3	4	5	6
Head	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
Tail	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

a)P(head)= $\frac{6}{12} = \frac{1}{2}$ b) P(2)= $\frac{2}{12} = \frac{1}{6}$

c) P(tail and a 5)= $\frac{1}{12}$ d) P(tail or a 5)= $\frac{7}{12}$

Exercise 4

1. Bag A contains one red disc and one blue disc. Bag B contains one red, one blue and one white disc. Draw a grid of the sample space when one disc is taken at random from each bag. Hence determine the probabilities of getting:
   a) two red discs
   b) two discs of the same colour
   c) a white disc
   d) two discs that are different colours

2. A coin is tossed and a spinner with six equal sectors marked 1, 2, 3, 4, 5 and 6 is twirled.
   a) Draw a grid of the sample space.
   b) How many outcomes are possible?
   c) Use your grid to determine the chance of getting:

i. a tail and a 3
ii. a head and a number greater than 4
iii. a tail and a number greater than 4
iv. a 6
v. no 6's
vi. any number except 5 and a head
vii. a head or a 6

Tree diagrams

Tree diagram can be used to illustrate sample spaces provided that the alternatives are not too many and then we are able to calculate probabilities.

Example:

During the holidays, the probability that Jane will play tennis on any day is $\frac{5}{7}$,

and the probability that she swims on any day is $\frac{3}{5}$.

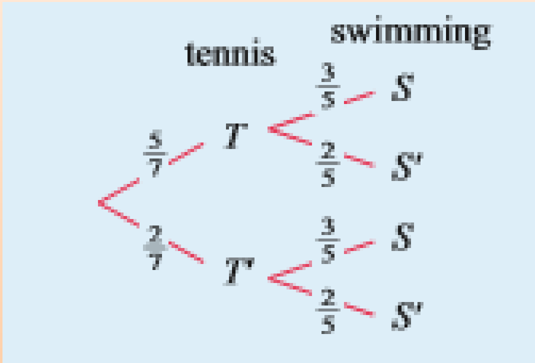
a) Draw a tree diagram to illustrate this situation
b) Use the tree diagram to determine the chance that on any day Jane:
  i) plays tennis and swims
  ii) swims but does not play tennis

Solution:

Let T represent the event 'Jane plays tennis and S represent the event 'Jane swims'

Thus, $P(T) = \frac{5}{7}$ and $P(T') = \frac{2}{7}$ $P(S) = \frac{3}{5}$ and $P(S') = \frac{2}{5}$

a)

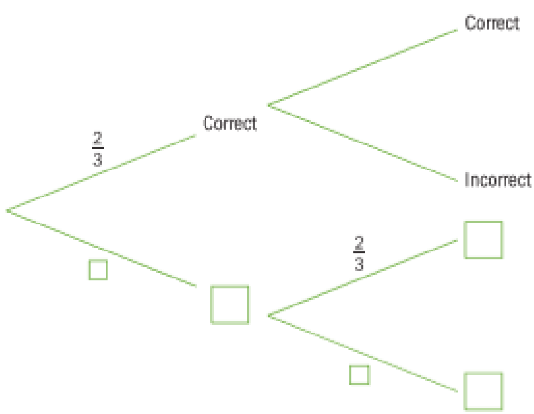


b) i) P(plays tennis and swims)= P(T and S)= $\frac{5}{7} \times \frac{3}{5} = \frac{3}{7}$

ii) P(swims but does not play tennis) = P(S and T') = $\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$

Exercise 6: To be discussed in pairs.

1. Lizzie is attempting two exam questions. The probability that she gets any exam question correct is $\frac{2}{3}$.
   a) Copy and complete the diagram.
   b) What is the probability that she will get only one of them correct?
   c) What is the probability she will get at least one correct?



2. When John and Mark play in the hockey team the probability that John scores is $\frac{1}{3}$ and that Mark scores is $\frac{1}{2}$.
Draw a tree diagram to illustrate this information and use it to find the probability that neither will score in the next game.

3. The probability of a day being windy is 0.6. If it is windy, the probability of rain is 0.4. If it is not windy, the probability of rain is 0.2.
   a) Draw a tree diagram to illustrate this information
   b) What is the probability of a given day being rainy?
   c) What is the probability of two successive days **not** being rainy?

4. There are equal numbers of boys and girls in a school and it known that $\frac{1}{10}$ of the boys and $\frac{1}{10}$ of the girls walk to school every day. Also $\frac{1}{3}$ of the boys and $\frac{1}{2}$ of the girls get a lift to school. The rest are boarders.
Determine, using a tree diagram,
   a) the fraction of the school proportion that are girls who are boarders.
   b) the fraction of the school population that are boarders.

5. Garang does his shopping at the same time every Friday. He estimates that the probability that he has to queue at the supermarket checkout is $\frac{5}{6}$, and the probability that he has to queue at the post office is $\frac{1}{4}$. When he does his shopping next Friday, what is the probability that he will queue at
   a) both places?
   b) neither places?

6. Nyoka has an 80% chance and Roger has a 50% chance of passing in their History examination. Find the probability that:
   a) both will pass
   b) Roger will pass and Helena will not
   c) neither will pass

7. In the month of November, the probability that it will rain tommorow is $\frac{2}{3}$, and the probability that I will carry my umbrella is $\frac{1}{4}$. What is the probabality that tomorrow:
   a) It will rain and that I will carry my umbrella
   b) It will rain and I will forget my umbrella
   c) It will not rain and I will forget my umbrella